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The Influence of Rainfall on the Yield of Wheat at Rothamsted

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III. *The Influence of Rainfall on the Yield of Wheat at Rothamsted.*

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Contents.

1. *General Problem of Evaluating the Effects of Weather on Crops.*

At the present time very little can be claimed to be known as to the effects of weather upon farm crops. The obscurity of the subject, in spite of its immense importance to a great national industry, may be ascribed partly to the inherent complexity of the problem which it presents, and more especially to the lack of quantitative data obtained either under experimental or under industrial conditions, by the study of which accurate knowledge alone can be acquired. Of the industrial applications of such knowledge it is unnecessary to speak here in detail. It is sufficient to indicate that the present system of Life Insurance, which safeguards the economic stability of many thousands of families, and occupies the activities of many of the greatest financial corporations, was made possible by the studies of statistics of human mortality by the mathematicians of the eighteenth and nineteenth centuries, and that on the basis of adequate knowledge similar economic stability with its attendant security of capital should be within the reach of the industrial farmer.

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The inherent complexity of the relationships which it is sought to elucidate, between the yields of farm crops, and the previous weather which largely controls those yields, arises primarily from the complexity of the problem of specifying the weather itself. Meteorologists have, however, gradually devised a number of instrumental observations, which although far from specifying the total environment of the growing plant, as understood by the plant physiologist, do nevertheless give a sufficiently detailed account of the general environmental conditions of the growing crop, in so far as these vary from season to season. It is probable indeed that almost all the weather influences to which crop variations are due could be expressed in terms of the instrumental observations of a modern meteorological station. The actual difficulty of calculating the crop variations from given instrumental records is, however, immense; only an attack of the most preliminary kind upon the general problem can be attempted in this paper. The complete aim of agricultural meteorology should, however, be emphasised, for it is only by its substantial achievement that other causes of crop variation can be freed from much obscurity.

A valuable example of an investigation in which this aim was clearly held in view is to be found in A. WALTER's study in 1910 of the effects of weather upon the Sugar crop in Mauritius (17). WALTER was able to give a very complete account of the meteorological causes affecting the yield of sugar, to such an extent that this author considers that under uniform estate management and methods of cultivation the yield may be predicted from meteorological data only with a standard error of the order of 1 per cent. The number of harvests available for this enquiry seems, however, to have been only 13, and more recent investigations in statistical theory make it now possible to realise, more clearly than was at the time possible, some of the dangers which arise from paucity of crop data.

2. *Effects of paucity of Crop Data.*

The biometrical investigations for which the method of multiple correlation was developed, differ from such agricultural studies as the present in two main particulars. In the first place the number of individuals measured was taken to be large, of the order of 1,000, or at least of some hundreds, and the special problems of distribution which arise in small samples have only more recently begun to receive attention. In the second place the number of measurements taken of each individual, or the number of *variates*, was generally small, and in all cases far smaller than the number of meteorological elements which may plausibly be regarded as affecting the crop. The sequence of weather to which crop variations may be ascribed extends over a year, or even more, and consists in this country of a series of abrupt, relatively violent and transient spells, each of which has its influence on the crop. If we wished to analyse the sequence no more closely than by monthly averages, we should still have 12 values for rainfall, and 12 more for maximum and minimum temperature, dew point, grass minimum, solar maximum and soil temperatures, nor would it be unreasonable to include some such measure of insolation as is given by "Hours of Bright Sunshine," and averages for the direction and force

of the wind. The number of meteorological elements might be made to exceed even the longest series of crop records available, for the Rothamsted wheat records provide, after necessary deductions, only 60 yields. Consequently, if the computer were provided not with yield data at all, but with an equal number of values composed at random, he would still be able to express them with perfect accuracy in terms of the weather records, for the number of unknowns available would exceed the number of equations for them to satisfy.

It is of more practical importance that even when we have selected a number of meteorological variables which is less than the number of crops recorded, a strong semblance of dependence may be produced, even when fictitious data, unrelated to the weather, are substituted for the true crops. For let there be n values of the dependent variate y and n each of the q meteorological variates $x_1, x_2, \ldots, x_q$. If now we obtain a partial regression equation

$$\mathrm{Y} = \bar{y} + \mathrm{S}\,(c_k\,(x_k - \bar{x}_k)),$$

showing the apparent dependence of y upon $x_1, \ldots, x_q$, then the correlation between y and Y, namely R, will be the multiple correlation of y with $x_1, \ldots, x_q$, and will be greater than the correlation of y with any other linear function of $x_1, \ldots, x_q$. Since R is necessarily positive (or zero), as was noticed by Hooker (10, p. 7), there will be generally an appearance of correlation produced in this way, even if no such relation really exists. This effect becomes particularly marked if the number in the sample is small.

When there is no real correlation it is possible to determine *a priori* the distribution of the multiple correlation of any quantity y with any q variates $x_1, x_2, \ldots, x_q$. There are many ways of approaching the solution, of which perhaps the most enlightening is to regard the deviations $y - \bar{y}$ as components determining the direction of a radius vector OP in n dimensions. Then q other directions are specified by the q meteorological variates, and any linear regression formula is represented by some line through the origin in the space of q dimensions, specified by these q directions. The multiple correlation, R, is then the cosine of the angle which the line OP makes with the space of q dimensions. If the variate y is unrelated to the variates $x_1, \ldots, x_q$, the line OP may be regarded as drawn at random through O, in the space of n dimensions; using this fact it may be shown without difficulty that the chance that R^2 falls into the elementary range $d\mathrm{R}^2$ is

$$df = \frac{\frac{n-3}{2}!}{\frac{n-q-3}{2}!\,\frac{q-2}{2}!}\,(\mathrm{R}^2)^{\frac{q-2}{2}}\,(1-\mathrm{R}^2)^{\frac{n-q-3}{2}}\,d\,(\mathrm{R}^2). \qquad \ldots\ldots\ (\mathrm{I})$$

It will be noted that the distribution is independent of any correlations which may exist between the meteorological variates, provided no one of them can be exactly

calculated from the others. In the case $q = 1$, when only one independent variate is used, the formula reduces to

$$df = \frac{2.\frac{n-3}{2}!}{\frac{n-4}{2}!\sqrt{\pi}}(1-R^2)^{\frac{n-4}{2}}dR,$$

which is known (FISHER (4), 1915) to be the distribution of the correlation (taken positive) derived from a sample of n, between two uncorrelated variates. It may also be seen that when $q = n - 1$, R is necessarily equal to $+1$.

The mean value of R^2 is

$$\overline{R^2} = \frac{q}{n-1},$$

and since

$$S(Y-\bar{y})^2 = R^2 S(y-\bar{y})^2,$$

we may say that in the absence of correlation, the variance of y (the square of its standard deviation) will be distributed on the average equally between the q degrees of freedom of the regression formula, and the $(n - q - 1)$ degrees of freedom in which y departs from the regression formula. This view of the matter enables us to follow the above conclusions to some extent into the more general case in which y is really in some degree correlated with $x_1, \ldots, x_q$. For if y be imagined as made up of two parts, $y_0 + y'$, such that y_0 is wholly determined as a linear function by the independent variates and y' is wholly uncorrelated with them, then y_0 and y' must be mutually uncorrelated, so that each will contribute a certain percentage to the variance of y. Let the fraction contributed by y_0 be A, then the regression formula found by correlating a sample of y' with the independent variates will differ from that found by correlating y, merely by y_0 which is by hypothesis a linear function of $x_1, \ldots, x_q$. Consequently the average proportion of the variance in the regression formula for y will be

$$\overline{R^2} = A + \frac{q}{n-1}(1-A),$$

or

$$1 - \overline{R^2} = (1-A)\frac{n-q-1}{n-1}.$$

If the sample be increased indefinitely $(n \to \infty)$ then the limiting value of R^2 is A. An estimate of A from a finite sample, free from the positive bias of R^2, is obtained by taking

$$1 - A_1 = \frac{n-1}{n-q-1}(1-R^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (II)$$

A complete discussion of the errors of random sampling in multiple regression requires a knowledge of the frequency distribution of R, or of A_1, calculated by equation (II), in the general case when A is not zero; equation I gives this distribution only when

$A = 0$. It provides a means of testing if an observed value of R differs significantly from zero, but not for testing in general, whether one value of R^2 is significantly greater than another.

From equation (I) it appears that the probability that R, obtained from a random sample of n observations, should exceed any specified value, is, if q is even,

$$P = (1 - R^2)^{\frac{1}{2}(n-q-1)} \left\{1 + \frac{n-q-1}{2} R^2 + \frac{(n-q-1)(n-q+1)}{2.4} R^4 + \ldots \frac{(n-q-1)\ldots(n-5)}{2.4\ldots(q-2)} R^{q-2}\right\},$$

and, if q is odd,

$$P = \frac{2}{\sqrt{\pi}} \frac{\left(\frac{n-q-2}{2}\right)!}{\left(\frac{n-q-3}{2}\right)!} \int_{\sqrt{\frac{R^2}{1-R^2}}}^{\infty} \frac{dz}{(1+z^2)^{\frac{1}{2}(n-q)}}$$

$$+ \frac{2}{\sqrt{\pi}} \frac{\left(\frac{n-q-2}{2}\right)!}{\left(\frac{n-q-3}{2}\right)!} R\,(1 - R^2)^{\frac{1}{2}(n-q-1)} \left\{1 + \frac{n-q}{3} R^2 + \ldots \frac{(n-q)(n-q+2)\ldots(n-5)}{3.5\ldots(q-2)} R^{q-3}\right\}.$$

The analogy of these formulæ with those giving the probability, P, that χ^2, the Pearsonian test of goodness of fit, should exceed any specified value is obvious. It will be noticed that in the second formula the probability integral of the normal curve is replaced by that of the Type VII curves which has been tabulated by "STUDENT" (16, 1917). When q is small, very few of the terms of the series are involved, thus for $q = 6$, or 7, the series terminates with the term in R^4.

The effect of increasing q, the number of variates used, is to increase somewhat rapidly the probability that R should exceed any specified value, even for an independent variate wholly uncorrelated with those used to predict it; this increase may be exemplified in the following table, where we have taken $n = 13$, $q = 4$, 6, 8 and calculated the chance of R exceeding 0·5, 0·6, 0·7, 0·8 and 0·9.

TABLE I.—$n = 13$.

q	R = 0·5	0·6	0·7	0·8	0·9
4	0·6328	0·4094	0·2002	0·0598	0·0055
6	0·8965	0·7491	0·5187	0·2509	0·0505
8	0·9844	0·9402	0·8248	0·5906	0·2424

It will be seen that the chance of obtaining a value $R > 0{\cdot}9$ is only one in 180 for $q = 4$; it rises to one in twenty for $q = 6$, and about one in four for $q = 8$. For $q = 8$, therefore, an observed multiple correlation $0{\cdot}9$ cannot be regarded as significant, that is as convincing evidence that the "crop" is in any way influenced by the meteorological variates. The only value, in fact, in the above table which could be regarded as definitely significant is the value $0{\cdot}9$ for $q = 4$, while for $q = 6$ this is suggestive only of real influence.

A still more insidious source of illusory high correlations lies in the fact that the particular variates chosen for correlation with the crop figures, are often chosen *because* they appear in fact to be associated with the crop. It is a common practice, as a preliminary to the study of weather correlations, to search for the so-called critical periods by such methods as the following. The years are arranged in order of crop yield, and a number of meteorological values are plotted on a chart in this order; those meteorological values which show an apparent trend upwards or downwards, are then picked out, and used to construct a weather formula by which the crop may be predicted. Such methods are especially deceptive when the series of years is short (about 20 or 30), for if such a process were carried out thoroughly we should not have merely the random sampling distribution of R, which as we have seen is capable of yielding sufficiently high values from uncorrelated material, but we should be choosing that set of q variates, out of a larger number p, which gave the highest value of R.

The effect of such a process, if dummy data were substituted for the actual crop records, could be accurately foretold by the solution of the following problem. A line OP is drawn through a point O, at random in n dimensions, p directions (where p may exceed n) are also chosen at random, and of them q ($q < n$) are selected so that the space of q dimensions through these makes the least possible angle, θ, with OP. Find the sampling distribution of θ (or, of $R = \cos\theta$).

I can put forward no general solution of this problem; the case $p = q$ has been solved above. The additional advantage conferred by a choice of variates may be clearly seen from the case $q = 1$. For

$$\int_0^R \frac{2 \cdot \frac{n-3}{2}!}{\sqrt{\pi} \cdot \frac{n-4}{2}!} (1 - R^2)^{\frac{n-4}{2}} dR$$

represents the chance that for a single uncorrelated variate chosen at random, the correlation in a sample shall not exceed R; hence it follows that the chance that for none of p uncorrelated variates chosen at random, does the correlation exceed R, must be

$$\left\{ \int_0^R \frac{2 \cdot \frac{n-3}{2}!}{\sqrt{\pi} \cdot \frac{n-4}{2}!} (1 - R^2)^{\frac{n-4}{2}} dR \right\}^p ;$$

and this is the chance that the highest correlation of p shall not exceed R.

If a standard of significance be chosen such that it will be exceeded in random samples once in 20 times, the effect of having a choice of p variates will increase the chance of exceeding any assigned rare value nearly p times. Thus for $n = 13$, 0·6 will not be judged significant, even for a single variate correlated if p exceed 2, 0·7 ceases to be significant if $p > 7$, and 0·8 when p exceeds about 54. When we consider that p may be very great, since not only is the number of meteorological elements available for selection large, but also since many writers allow themselves to use complicated functions of the instrumental data, involving adjustable weights, special conventions of sign, and allowances for periods judged to be unusual in their effects, so that these artificial meteorological data may be calculated in an enormous variety of ways, it is clear that the conclusions we have drawn as to the dangers of applying multiple correlation formulæ to small samples, are very much to be emphasised when a choice is made as to what meteorological elements to correlate.

In view of the foregoing facts it would seem worth while to lay down the following conditions for arriving at unprejudiced results :—

(i) The meteorological variates to be employed must be chosen without reference to the actual crop record.

(ii) If multiple variates are to be used allowance must be made for the positive bias of R^2.

(iii) Relationships of a complicated character should be sought only when long series of crop data are available.

3. *The Analysis of the Season.*

By the Season is meant the whole sequence of weather directly or indirectly influencing the crop from its inception to the time the produce is weighed. In studying the influence of the rain on the wheat crop, we have chosen a period of a year ending on the 31st of August of the year in which the crop was harvested. To obviate the irregularity of the calendar, this year was taken to be of 366 days, so that it commenced on either the 31st of August or the 1st of September prior to the sowing of the seed. Taking into consideration a single element, namely Rain, only, this sequence is in our climate sufficiently complex. The general distribution of rain through the year could, indeed, be roughly represented by dividing the period up into say 6 sections and recording the total rainfall in each period. If the rainfall were approximately evenly distributed in successive days or weeks, this method might indeed give a fairly accurate picture of the sequence of rainfall. But notoriously the rain is in fact often concentrated in short spells, with rainless periods intervening, consequently if March and April formed one section, a great part of the rain ascribed to that section might fall early in March in one year, and late in April in another, and it would be impossible to regard such falls as equivalent

in their effects on the crop. Whereas a late April fall might well be nearly equivalent to one occurring early in May.

This consideration suggests that a finer subdivision of the year is necessary; that months or even weeks should be treated separately. From the agricultural and meteorological point of view there is everything to be said for such subdivision; it raises, however, new mathematical difficulties; for whereas the evaluation of determinants of 6 rows and columns, or the solution of simultaneous linear equations in 6 unknowns is sufficiently rapid, disproportionate labour is involved in increasing the 6 to 12, and if 52 unknowns were attempted the labour involved would become fantastic.

A consideration which has been still more influential in framing our method is that even if we were to calculate the 52 partial regression coefficients showing the average effect in bushels per acre of an extra inch of rainfall, for each week of the year, such a calculation would leave out of consideration the all-important fact that this effect may be expected to change *continuously* during the year. The true partial regression coefficients for neighbouring periods must be relatively alike. The method of partial correlation as hitherto developed, takes no account of the serial character of our weather variates; after calculating the partial regression coefficients, we should still be far from the facts, if we did not smooth the series so obtained by a continuous curve, which should average out the independent errors in the values obtained for the successive weeks.

Disregarding, then, both the arithmetical and the statistical difficulties, which a direct attack on the problem would encounter, we may recognise that whereas with q subdivisions of the year, the linear regression equations of the wheat crop upon the rainfall would be of the form

$$\bar{v} = c + a_1 r_1 + a_2 r_2 + \ldots + a_q r_q$$

where $r_1, r_2, \ldots, r_q$ are the quantities of rain in the several intervals of time, and $a_1, \ldots a_q$ are the regression coefficients, so if infinitely small subdivisions of time were taken, we should replace the linear regression function by a *regression integral* of the form

$$\bar{v} = c + \int_0^T a r \, dt, \qquad \text{(III)}$$

where $r\,dt$ is the rain falling in the element of time dt; the integral is taken over the whole period concerned, and a is a *continuous* function of the time t, which it is our object to evaluate from the statistical data.

It will be seen that by proceeding to the limit all artificiality has been eliminated from the quantity a, which now represents an objective physical quantity, namely the average benefit to the crop in bushels per acre per inch of rain, falling in the time-element considered. This, of course, is more than even a daily record of rain can tell us, but owing to the relatively slow changes in the functions a, we shall find it sufficient to divide the 366 day year into 61 equal periods of 6 days each.

It should be noted that corresponding to the quadratic terms of a regression formula the independent variates of which form a continuous series, *i.e.*, to

$$\mathrm{S}\,(b_{pq}r_p r_q),$$

we have the double regression integral

$$\int_0^{\mathrm{T}}\int_0^{\mathrm{T}} b\,(t,\,t')\,r_t r_{t'}\,dt\,dt',$$

where b is a continuous function of the two epochs t and t', just as a is a continuous function of t only.

The concept of the regression integral, involving a regression function varying continuously with the time, is not only of service in displaying in a precise mathematical form the nature of the relationship which is to be investigated, but when the functions a and b vary with the time relatively slowly, it suggests a statistical procedure by which values of these functions may be obtained from the data. That the values must change relatively slowly is apparent from the fact that the state of advancement of the crop, as indicated either by the time of harvest, or by phenological observations on common weeds, often varies relatively to the Calendar date by as much as a fortnight.

If, then, T_0, T_1, T_2 ... be a series of orthogonal functions of the time, such that

$$\int_0^{\mathrm{T}} \mathrm{T}_r\mathrm{T}_s\,dt = 0 \qquad (r \neq s)$$

$$\int_0^{\mathrm{T}} \mathrm{T}_r^2\,dt = 1,$$

we may express the rate of rainfall at any epoch, in the form

$$r = \rho_0\mathrm{T}_0 + \rho_1\mathrm{T}_1 + \rho_2\mathrm{T}_2 + \ldots,$$

where

$$\rho_s = \int_0^{\mathrm{T}} r\mathrm{T}_s\,dt;$$

also we may express the regression function in the form

$$a = \alpha_0\mathrm{T}_0 + \alpha_1\mathrm{T}_1 + \alpha_2\mathrm{T}_2 + \ldots, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IV)}$$

where

$$\alpha_s = \int_0^{\mathrm{T}} a\mathrm{T}_s\,dt,$$

noting in the latter case that relatively few terms of the expansion will be required, if a is a slowly-varying quantity.

Then we shall have from equation (III)

$$\bar{w} = c + \int_0^T ar\,dt = c + \alpha_0\rho_0 + \alpha_1\rho_1 + \alpha_2\rho_2 + \ldots .$$

Now the values of ρ_0, ρ_1, ρ_2, ..., may be obtained for each year from the rain record, and by correlating them with the crop data, we can obtain the values of α_0, α_1, α_2, ... , as partial regression coefficients of the crop on the coefficients of the rainfall distribution ; then from (IV) we can evaluate the expansion of a to as many terms as we have obtained. It is easy to verify that in the same manner the double regression integral may be expressed as a quadratic regression formula for the crop in terms of the rainfall distribution values.

The advantage of this indirect method of attack lies not only in the facts that for a fine division of the year, the arithmetical difficulties of a direct attack are insurmountable, and that no existing crop record is sufficiently long to justify the calculation from it of 50 or 60 independent regression coefficients, for the probable error of such determinations depends on the excess of the number of observations over that of the coefficients evaluated (9), but also in the increased accuracy attained through utilising our knowledge that the regression function must in fact vary relatively slowly. This advantage is analogous to that obtained by the author in collaboration with Miss Mackenzie, who found that using the correlations of weekly rainfall between different stations in Great Britain from a record of about 40 years, it is possible to estimate the correlation, for any one week of the year, with an accuracy which, for any single week, would have required records for over 1,000 years ; this was due to the fact that the sequence of correlations could be well represented by a harmonic curve of only the second order, so that use could be made of some 2,000 actual simultaneous observations of weekly rainfall to eliminate errors of random sampling (8. Fisher and Mackenzie, 1922). In the present investigation the accuracy is limited by the number of crops recorded, but the method employed enables full use to be made of the meteorological data.

In the practical application of the method of the regression integral, it is not necessary that the rainfall record should be strictly continuous ; indeed daily observations are themselves too numerous to be conveniently handled. The rain was therefore divided up into 61 periods of 6 days each, the integrations being replaced by summations over 61 terms. This introduces a slight modification into the form of the orthogonal functions of the time, for it is necessary for exact work that

$$\sum_1^{61} (T_r T_s) \qquad r \neq s$$

should vanish. The most convenient orthogonal functions to use are those developed by Esscher (3) in respect to mortality, and independently by the present author, in eliminating the slow changes observable in the wheat yields of the experimental plots

(5, 1921). If the time is measured from the middle point of a series of n terms, in units equal to the interval between successive terms, we have

$$T_0 = \frac{1}{\sqrt{n}},$$

$$T_1 = \sqrt{\frac{12}{n(n^2-1)}}\,t,$$

$$T_2 = \sqrt{\frac{180}{n(n^2-1)(n^2-4)}}\,(t^2 - n_2),$$

$$T_3 = \sqrt{\frac{2{,}800}{n(n^2-1)(n^2-4)(n^2-9)}}\left(t^3 - \frac{n_4}{n_2}t\right),$$

$$T_4 = \sqrt{\frac{44{,}100}{n(n^2-1)(n^2-4)(n^2-9)(n^2-16)}}\left(t^4 - \frac{n_6 - n_2n_4}{n_4 - n_2^2}t^2 + \frac{n_2n_6 - n_4^2}{n_4 - n_2^2}\right),$$

$$T_5 = \sqrt{\frac{698{,}544}{n(n^2-1)(n^2-4)(n^2-9)(n^2-16)(n^2-25)}}\left(t^5 - \frac{n_2n_8 - n_4n_6}{n_2n_6 - n_4^2}t^3 + \frac{n_4n_8 - n_6^2}{n_2n_6 - n_4^2}t\right),$$

where n_2, n_4, n_6 and n_8 stand for the mean values of t^2, t^4, t^6 and t^8. So that

$$n_2 = \tfrac{1}{12}(n^2-1),$$

$$\frac{n_4}{n_2} = \tfrac{1}{20}(3n^2-7),$$

$$\frac{n_6 - n_2n_4}{n_4 - n_2^2} = \tfrac{1}{14}(3n^2-13),$$

$$\frac{n_2n_6 - n_4^2}{n_4 - n_2^2} = \tfrac{3}{560}(n^2-1)(n^2-9),$$

$$\frac{n_2n_8 - n_4n_6}{n_2n_6 - n_4^2} = \tfrac{5}{18}(n^2-7),$$

$$\frac{n_4n_8 - n_6^2}{n_2n_6 - n_4^2} = \tfrac{1}{1008}(15n^4 - 230n^2 + 407).$$

These functions are in reality very convenient to handle, the practical arithmetic involved will be explained in Section (5) under the analysis of the rain data. If n be increased the functions tend to take the form of Legendre Polynomials.

4. *The Correlation of Residuals.*

When it is desired to study the correlation of variables such as annual figures, in which progressive changes are observable, it is necessary in order to obtain results which can be relied upon, and compared, in the same manner as those obtained from homogeneous material, to eliminate in some way the influence of the progressive changes. Material of this kind is very abundant and of the utmost importance ; the bulk of official statistics,

vital, economic, epidemiological, meteorological and agricultural, may be said to be awaiting a method of reduction which shall deal adequately with the difficulties which this type of data presents. My investigations have not led me to any satisfactory or complete solution of the problem, but since systematic methods have been developed and strongly advocated, it would seem worth while at the present time to put forward a group of mutually connected considerations, which shed new light upon various aspects of the problem, and bring clearly into view the sources of error to which the more obvious methods of approach are exposed.

It has been observed (4, FISHER, 1915) that if the individual values of a sample be taken as the co-ordinates of a point in generalised space, the mean standard deviation and coefficient of correlation of a sample are capable of a very beautiful geometrical interpretation. For if we assign any value $\bar{x}$ as the mean of n observations $x_1, x_2, \ldots, x_n$, the equation

$$\mathrm{S}(x) = n\bar{x}$$

represents a plane section of the distribution, placed at right angles to the line

$$x_1 = x_2 = \ldots x_n$$

and meeting it at the point

$$x_1 = x_2 = \ldots = x_n = \bar{x}.$$

If this point is taken as origin, we have

$$\bar{x} = 0,$$

and the point representing any sample has rectangular co-ordinates

$$x_1 - \bar{x}, x_2 - \bar{x}, \ldots, x_n - \bar{x},$$

so that its distance from the new origin is the square root of

$$n\mu^2 = \mathrm{S}(x - \bar{x})^2$$

where μ is the standard deviation of the sample.

The mean of the observations thus specifies the origin, and the standard deviation specifies the length of the radius vector from the origin to the sample point; the correlational properties of the sample depend on the direction of this radius vector. For if a corresponding construction be made for the values of a second variate

$$y_1, y_2, \ldots, y_n,$$

and each of the axes of the y-space be brought into coincidence with the corresponding axes of the x-space, it has been shown, and it is easy to see, that the correlation coefficient between x and y is the cosine of the angle between the two *radii vectores*.

The geometrical interpretation of the correlation coefficient provides the simplest possible proof of a theorem of some importance in the present connection, namely, that: *If x_1', x_2', ... x_n' be the co-ordinates of the first sample-point with respect to any rectilinear*

orthogonal axes through the same origin and y_1', y_2', ... y_n' *the corresponding functions of* y_1, y_2 ... y_n *then the correlation coefficient between* x' *and* y' *is equal to the correlation coefficient between* x *and* y.

The system of orthogonal functions of the chief practical importance are those which arise in the elimination of progressive changes by means of a polynomial function of the time. These are the same functions as are used in analysing the sequence of weather in each particular season; it is worth noting that the above theorem is true of any orthogonal system.

In an unchanging series of independent values, *i.e.*, one in which the frequency of occurrence of any value is independent of the time, all the quantities x', except x_1', will be distributed in symmetrical curves about the mean value zero. If the values of x are normally distributed about the mean m, with standard deviation σ, then all the values of x' will be normally distributed with standard deviation σ, x_1', about a mean $m\sqrt{n}$, and the remainder about zero. This follows at once from the fact that the distribution of the points representing the sample will be a globular normal cluster, and consequently the distribution of their projections on a line drawn in any direction will be invariable. Even when the values of x are not normally distributed, those of x' will in general be much more nearly so, owing to the properties of compound distributions which have often been discussed.

The simplest type of changing series is one in which the several values are distributed independently and in similar distributions, but about a changing mean. It is with a view to eliminating slow changes in the mean, that most methods of obtaining residuals from smoothed values are employed. It is usually the case that slow changes in the mean value may be sufficiently represented by a polynomial whose degree is small compared to the number of observations in the series. In such cases the first few values of x' will be abnormally great (positive or negative), while the remainder will be normally and independently distributed. The whole group of values of x' is heterogeneous, and the correlation of two such series will have no easy interpretation; its value may be governed by the values of a few exceptional terms, and therefore its probable error will be excessive. If, however, we reject the exceptional terms and calculate the correlation from the normally distributed remainder, we shall be correlating homogeneous material and the coefficient obtained will have a simple interpretation and its usual precision. Thus if the first p values of x' suffice to specify the slow changes in the mean value, the correlation required is that obtained by ignoring the first p values of x', and y', or in other words putting them equal to zero.

This process is only an extension of the universal practice of eliminating the mean values of the variates in calculating the correlation coefficient; in working with an unchanging series we habitually ignore the values of x'_1 and y'_1. In the more general case we take

$$(n-p)\mu_1{}^2 = \sum_{p+1}^{n}(x'^2), \qquad (n-p)\mu_2{}^2 = \sum_{p+1}^{n}(y'^2), \qquad (n-p)\mu_1\mu_2 r = \sum_{p+1}^{n}(x'y').$$

From another point of view this may be regarded as the result of finding the partial correlation between x and y when the first $(p-1)$ powers of t, regarded as a system of correlated variates, are eliminated.

The value of r obtained in this way is, as we have shown, identical with that obtained from the mean squares and mean products of the residuals of the two series after fitting polynomials of degree $(p-1)$. The values of such residuals are indeed heterogeneous in respect of variability, and mutually connected by p equations; but since the values of x' (and equally of y') are independent and equally variable, the value of r obtained from such residuals will be distributed in random samples, exactly as is that obtained from an ordinary correlation of $n-p+1$ independent homogeneous values. The frequency distribution of r in such cases has been treated in detail (6, FISHER, '*Metron*,' 1921).

The elimination from data of obvious heterogeneity is, of course, a different process from a process which leaves the material necessarily homogeneous. The latter process is perhaps unobtainable, but it would be desirable, in cases where complications are expected, to have a means of testing whether the heterogeneity has been sufficiently eliminated. The elimination of the slowest changes is easy, since the earliest terms of the polynomial are quickly calculated. The size of the coefficients affords a criterion of rejection; thus for a record of the dressed grain from two plots on Broadbalk for 67 years I obtained (5, 1921).

Dunged plot 2B.			Plot 11 (no potash).		
Degree.	x'/μ,	P.	Degree.	x'/μ.	P.
1	−0·82	0·41	1	−4·78	0·0000018
2	−2·38	0·017	2	+0·51	0·61
3	−2·50	0·012	3	+0·20	0·84
4	−2·06	0·039	4	−0·96	0·34
5	+4·02	0·000058	5	+1·72	0·086

This first column shows the degree of the polynomial term; the second, the value of x' in terms of the standard deviation as estimated from the residuals after a curve of the 5th degree had been eliminated; the third column gives the probability of such a term occurring by chance.

Since the standard deviation is derived effectively from the sum of 61 independent squares, the exact form of the distribution of x/μ, which is STUDENT'S Type VII curve, for $n = 62$, is taken to be equivalent to the normal distribution. On the dunged plot the first term gives no grounds for elimination; no significant deterioration having occurred on this plot, and it is immaterial whether or not such a term were retained in the correlations. The remaining four terms all show the existence of a slow change

in the yield. These four terms contribute to the variance over half as much as the remaining 61 terms put together. It should be noted that while little can be lost in rejecting the first term from the correlation, nothing can be gained by retaining it, for though its smallness renders it innocuous, its claim to rank as part of the homogeneous material is overturned by the size of the following terms. On plot 11 the deterioration due to exhaustion of potash is very great, and no other term except perhaps the 5th is large enough to excite suspicion. Our grounds for eliminating the first five terms in this case are that all the plots show similar and almost proportional slow changes, and that this is doubtless the case with Plot 11, where, however, the great variability of the annual yields has masked the significance of the coefficients.

Certainty as to the homogeneity of the remaining values of x' could, I think, only be obtained by a complete calculation of their values; in the absence of tables of the coefficients this would be very laborious. The whole set should be normally distributed about zero, and heterogeneity would be indicated by β_2 significantly exceeding its normal value, 3. In the neighbourhood of the normal curve β_2 has been shown to be the most efficient statistic for such a test (7, 1922); a method of calculating β_2 for the x' distribution from the actual values of x would therefore be a valuable resource, especially if any way could be found of calculating it without the separate evaluation of the values of x'. It is believed that little heterogeneity remains in the series of wheat yields from Broadbalk after fitting polynomials up to the 5th degree.

The occurrence of series of values exhibiting slow changes is so widespread that it is not surprising that a number of different methods have been prepared for dealing with the difficulty, and that some difference of opinion should exist as to the disadvantages of each. We shall show that all the methods of elucidation which have been at all fully developed fall into one class, and that the main drawback of this class of methods is reduced to a minimum by the use of polynomials fitted to the whole series.

Slow changes may be eliminated in several ways :—

(i) by fitting a polynomial or other curve to the whole of the data ;
(ii) by the use of " smooth " values obtained by compounding a number of neighbouring terms ;
(iii) by repeatedly differencing the observed series.

In all these cases the resulting residuals designed for use as correlation variates may be successfully freed of any slow progressive trend which vitiates the original series; but in all cases this is achieved by entangling together to some extent the successive values. This effect is inevitable, for we must judge of the smooth value from which our residual is measured by the values at neighbouring epochs.

It might appear that simple differencing should be placed in a separate class, since here no smooth value objectively appears. There is no essential difference however; for if $2r$ is any even integer, we can construct a function,

$$v = u - (-)^r k\delta^{2r}u,$$

which is in effect the smooth value, the deviations from which are proportional to the $(2r)^{\text{th}}$ differences; to make $\overline{v^2}$ a minimum for a series of equally variable quantities in random order we require

$$k = \frac{(2r!)^3}{(r!)^2\, 4r!}.$$

Thus the smooth value corresponding to 6th difference is

$$\begin{aligned} v_0 &= u_0 + {}_{2}^{5}{}_{3}{}_{1}\,\delta^6 u_0 \\ &= {}_{2}^{1}{}_{3}{}_{1}\,\{131u_0 + 75\,(u_1 + u_{-1}) - 30\,(u_2 + u_{-2}) + 5\,(u_3 + u_{-3})\}. \end{aligned}$$

The formula so obtained is identical with that given by SHEPPARD (15, p. 31), for 7 point smoothing, using a polynomial of the 5th degree. The variate difference method is therefore only an extreme form of the use of SHEPPARD'S smoothing formula—the extreme in which the number of terms is a minimum for given degree of the slow change eliminated—while the other extreme is represented by the process of fitting a polynomial of the required degree to the whole of the series.

In treating these three processes as special forms of SHEPPARD'S smoothing process one distinction must be made. In fitting a polynomial to the whole of the series we wish to use not only the residual of the middle term, but the whole series of residuals. So in using smooth values from (say) a 15 point formula, the first and last seven residuals may be obtained from the curves fitted to the first and last sets of 15 points. In the applications which have been made of the Variate Difference Method, only the residuals of the middle terms have been used. The number in the series has been diminished by one with each differencing; if however we wish to add the missing terms, this is easily done by means of binomial coefficients; for example if

$$a,\ b,\ c$$

are the sixth differences of a series, the three missing residuals prior to a are as follows

$$-\frac{a}{20},\quad +\frac{6}{20}a,\quad -\frac{15}{20}a,\quad a,\ b,\ c,\ \ldots\ .$$

The effects of such processes in entangling the neighbouring terms may best be seen by considering the effect of applying them to an unchanging series of equally variable quantities. If u stand for such a quantity then, for example, the sixth difference may be written

$$v = -20u_0 + 15\,(u_1 + u_{-1}) - 6\,(u_2 + u_{-2}) + (u_3 + u_{-3}),$$

hence evidently

$$\begin{aligned} \overline{v^2} &= +924\,\overline{u^2}, \\ \overline{v_p v_{p+1}} &= -792\,\overline{u^2}, \\ \overline{v_p v_{p+2}} &= +495\,\overline{u^2}, \end{aligned}$$

and so on, the numerical coefficients being those of the expansion $(1+x)^{12}$. Consequently the correlations between neighbouring values of v will be

$$\begin{aligned} r_1 &= -\tfrac{6}{7} &&= -0{\cdot}8571, \\ r_2 &= \tfrac{5}{8}\cdot\tfrac{6}{7} &&= +0{\cdot}5357, \\ r_3 &= -\tfrac{4}{9}\cdot\tfrac{5}{8}\cdot\tfrac{6}{7} &&= -0{\cdot}2381, \\ r_4 &= &&= +0{\cdot}0714. \\ r_5 &= &&= -0{\cdot}0130, \\ r_6 &= &&= +0{\cdot}0011. \end{aligned}$$

If such high correlations as these are produced in an originally uncorrelated series, it is clear that they cannot be used without drastic correction in an examination into such correlations between neighbouring terms as exist in the original series. Equally large are the effects when two separate series are correlated. If we take, for example, a second series u', and obtain

$$v' = \delta^6 u',$$

then if the original two series were correlated so that

$$\overline{u_p u'_{p+k}} = \rho_k \sqrt{\overline{u^2}\,.\,\overline{u'^2}},$$

it is easy to see that the correlations obtained from the 6th differences will be expressible in terms of the ρ series, in the form

$$r_k = {}_{9}\tfrac{1}{2}{}_{4}\,\delta^{12}\rho_k.$$

By such a method therefore we shall not obtain the true values ρ_k, but quantities proportional to the 12th difference of the series. Only if all values of ρ, except one, vanish, and we correlate the corresponding values of u and u', will we obtain an estimate of the correlation unaffected by gross inaccuracy. This constitutes a fatal objection to the applications which have been made of the variate difference method in its original form.

The same source of error still persists in more moderate degree when smoothing formulæ involving more terms are used. Let us take for example SHEPPARD'S formula for fitting a polynomial of the 5th degree to sets of 15 points. The smoothed middle point is

$$\left(1 - {}_{11}^{50}\,\delta^6 + {}_{13}^{75}\,\delta^8 + {}_{13}^{36}\,\delta^{10} + {}_{17}^{19}\,\delta^{12} + {}_{3}^{1}{}_{7}{}_{3}^{5}\,\delta^{14}\right) u\,;$$

whence we have the residual

$$v = \frac{1}{11\,.\,13\,.\,17\,.\,19}\{35126u_0 - 10125(u_1+u_{-1}) - 7500(u_2+u_{-2}) - 3755(u_3+u_{-3}) + 165(u_4+u_{-4}) - 2937(u_5+u_{-5}) + 2860(u_6+u_{-6}) - 2145(u_7+u_{-7})\}.$$

The correlations between neighbouring terms of an originally uncorrelated series now become

$$\begin{aligned} r_1 &= -0{\cdot}3075 & r_8 &= -0{\cdot}0132 \\ r_2 &= -0{\cdot}2370 & r_9 &= +0{\cdot}0072 \\ r_3 &= -0{\cdot}1119 & r_{10} &= +0{\cdot}0158 \\ r_4 &= +0{\cdot}0355 & r_{11} &= +0{\cdot}0099 \\ r_5 &= +0{\cdot}1440 & r_{12} &= -0{\cdot}0027 \\ r_6 &= +0{\cdot}1211 & r_{13} &= -0{\cdot}0076 \\ r_7 &= -0{\cdot}1565 & r_{14} &= +0{\cdot}0028. \end{aligned}$$

The correlations are now much more moderate. Their alterations are less violent, and since their sum is compelled to be $-0{\cdot}5$, their actual values are permitted to be considerably smaller. Such values are, however, sufficiently large to show that great inaccuracy will be introduced if we ascertain the mutual correlations of members of a series from those of the residuals from smooth values obtained from 15 adjacent points.

It is evident also that, apart from the question of the correlation of successive values, the correlations between two series will be vitiated in the same manner as by the variate difference method, though to a less degree; the correlations obtained may be expressed in terms of high differences of the true correlations, for if we write

$$\phi(\delta^2) = \tfrac{5}{11}\delta^6 + \tfrac{7}{13}\tfrac{5}{13}\delta^8 + \tfrac{3}{13}\tfrac{6}{13}\delta^{10} + \tfrac{1}{17}\tfrac{0}{17}\delta^{12} + \tfrac{1}{3}\tfrac{5}{2}\tfrac{5}{3}\delta^{14},$$

and

$$\phi^2(\delta^2) \text{ for } \{\phi(\delta^2)\}^2,$$

then we have

$$r_k = \frac{\phi^2(\delta^2)\,\rho_k}{\phi^2(\delta^2)\,\rho_0}.$$

The general problem of eliminating the cross correlations from a pair of series showing slow changes is extremely complex; in the simplest case each value of the one series is correlated with only two adjacent values of the second series. To this case probably belongs the relation of wheat crop to weather, for the crop is admittedly much affected by the weather in the harvest year, and may to a much less extent be influenced, through the condition of the seed, or of the soil, by the weather of the previous year. It is fortunate, therefore, that one of the examples, upon which the largest amount of work has been expended, is probably also of the same type, and will serve to indicate the order of magnitude of the errors to be expected, through neglecting the cross correlation of the crop with the weather of the previous year.

The two series of infantile death rates in the first and second years of life, are probably connected principally, if not wholly, by the fact that the mortality, in any one year, of children in the second year of life, refers to nearly the same group of children as the mortality in the previous year of children in the first year of life; and by the second fact that mortality in the two age groups during the same year will be conditioned by the

same meteorological and epidemiological conditions. If the first effect is the main object of study, the latter will appear as a cross correlation introducing errors into our estimate of the first effect, according to the method of estimation employed.

In 1915 (1) ELDERTON and PEARSON found, using the variate difference method, the value $-0\cdot688$ for the correlation between the mortalities of the same group of children (males) in the two years. In 1923 the same authors using SHEPPARD'S 15-point smoothing formula find the value $-0\cdot463$. The discrepancy is very great and suggests that the neglect of the correlation between the mortalities of the two groups of children in the same year has produced a large negative bias, which is more pronounced in the earlier estimate. Fitting a polynomial to the whole series further diminishes, though it does not eliminate, the error; for the polynomials of the 4th, 5th and 6th degrees we find the values $-0\cdot308$, $-0\cdot311$, $-0\cdot377$.

Now, assuming that only two correlations are really operative, these values may be corrected by calculating in a similar manner the apparent correlations in mortality for the two groups of children in the same year. These values are $+0\cdot4706$, $+0\cdot5456$ and $+0\cdot5217$; taking, then, the correlation between successive residuals of a series of n terms fitted by a polynomial of degree r to be $-(r+1)/(n-1)$, we have the equations

$$\text{4th degree} \begin{cases} \rho_0 - 0\cdot0877\rho_1 = -0\cdot3083 \\ -0\cdot0877\rho_0 + \rho_1 = +0\cdot4706 \end{cases} \qquad \rho_0 = -0\cdot2691;$$

$$\text{5th degree} \begin{cases} \rho_0 - 0\cdot1053\rho_1 = -0\cdot3109 \\ -0\cdot1053\rho_0 + \rho_1 = +0\cdot5456 \end{cases} \qquad \rho_0 = -0\cdot2563;$$

$$\text{6th degree} \begin{cases} \rho_0 - 0\cdot1228\rho_1 = -0\cdot3768 \\ -0\cdot1228\rho_0 + \rho_1 = +0\cdot5217 \end{cases} \qquad \rho_0 = -0\cdot3175.$$

The values of ρ_0 obtained from these equations are unbiassed estimates of the correlation required; they agree in indicating a correlation about $-0\cdot3$. This value may be confirmed from the figures given by PEARSON and ELDERTON for their 15-point smooth curve, which lead to the equations

$$\begin{aligned} \rho_0 - 0\cdot3075\rho_1 &= -0\cdot4548 \\ -0\cdot3075\,\rho_0 + \rho_1 &= +0\cdot6521 \end{aligned} \qquad \rho_0 = -0\cdot2808.$$

The concordance of these results indicates that whereas the variate difference method has exaggerated the value of this correlation to the extent of more than doubling it, the correlation of residuals from the 15-point smooth curve has reduced the error to about 50 per cent., and is moreover capable of correction provided only a few important correlations are present. The method of polynomial fitting has introduced errors of about 20 per cent. only, and the correction applied to it may be expected to be for this reason all the more precise.

In calculating the effects of weather upon the crop, it is probable, therefore, that the

effects of weather previous to the harvest year considered may be ignored. Hooker (10, 1907) has indeed indicated that some real effect may be ascribable to previous weather, but his correlations in the case of wheat are low, and as we have seen, if the polynomials fitted to the whole series are utilised to eliminate the slow changes, the spurious correlations introduced by this cause must be much further diminished. In the case of the infantile death rates when the correlation ignored was much greater than that which we sought to evaluate, the value obtained was raised only from $-0{\cdot}26$ to $-0{\cdot}31$; in the present case we may be certain that the ignored effects are much smaller than those evaluated, and their effect, if any, on the values obtained must be extremely small.

5. *The Rothamsted Rain and Wheat Data.*

The Rothamsted wheat data employed in this investigation are derived from Broadbalk field, which has grown wheat under experimental conditions since 1844. Thirteen plots have been continuously under uniform treatment since 1852. This series is unique in its length, which as we have seen is a most important consideration for statistical purposes. A second feature of great value is that the yields are derived from measured plots by actual weighing, and are not derived from estimates based on visual observation as are the county averages published by the Ministry of Agriculture. An account of the variations in yield of dressed grain of these 13 plots has already been published (5, Fisher, 1921). It was found that the variations observable could be divided into three groups, ascribable to three separate causes. (i) On many of the plots a progressive diminution is observable owing to the exhaustion of the soil in certain of the essential plant nutrients; (ii) on all the plots slow changes in yield have taken place, which may probably be ascribed to variations in the weed infestation of the field as a whole; this variation, unlike that ascribable to other causes, is in all the plots approximately proportional to the mean yield. These two causes of variation may be eliminated by fitting to each series a polynomial of the 5th degree. The remaining variation, consisting of the deviations of the actual yield from the smooth average value given by the polynomial, is ascribable primarily to variations of the season under which the crop grew to maturity. Conventionally this season has been taken to be the weather for 366 days, ending on August 31st of the harvest year. The first two causes of variation have already been discussed; it is the purpose of the present paper to consider the fluctuations in yield from year to year in relation to the rainfall record.

Table II gives the manurial treatment of the 13 plots considered together with the mean yields obtained from them, and the mean rate of deterioration. In Table III are shown the deviations in each year from the corresponding polynomial value; it will be observed that certain years have been omitted as not available for correlation with rainfall. The rainfall record commenced in February, 1853, and is therefore complete for the crop harvested in 1854 and subsequently. In 1889 and 1890 the field was partially fallowed by halves, and the crops in 1890 and 1891 following the fallow were

beneficially affected; the same is true of the harvests of 1905, 1906 and also of 1915. It has been possible to use the value for 1916, since in this case the two halves of the field were harvested separately, and the half fallowed in 1915 could be rejected. With these omissions it is believed that the series of crop yields from this field are as homogeneous a series as it would be possible to obtain; it will be seen that 60 years are available for correlation with rainfall.

TABLE II.

Plot.	Manure per acre. Sulphate of potash.	Sulphate of soda.	Sulphate of magnesia.	Super-phosphate.	Sulphate of ammonia.	Chloride of ammonia.	Mean (Bushels per acre).	Mean annual diminution (Bushels per acre).
	lb.	lb.	lb.	lb.	lb.	lb.		
2B	Dung 14 tons		—	—	—	—	34·549	0·031
3 & 4	No Manure		—	—	—	—	12·269	0·097
5	200	100	100	392	—	—	14·180	0·090
6	200	100	100	392	100	100	22·581	0·141
7	200	100	100	392	200	200	31·367	0·144
8	200	100	100	392	300	300	35·694	0·092
10	—	—	—	—	200	200	19·504	0·157
11	—	—	—	392	200	200	22·046	0·219
12	—	366½	—	392	200	200	28·319	0·181
13	200	—	—	392	200	200	30·209	0·123
14	—	—	280	392	200	200	27·765	0·231
17 *	200	100	100	392	—	—	14·510	0·092
18 *	—	—	—	—	200	200	29·006	0·114

* Alternate.

In February, 1853, readings of rainfall were commenced with a large gauge, 0·001 acre, built for the purpose; the readings of this large gauge have been consistently higher than those of the 5-in. and 8-in. gauges which are at present placed beside it. It may be concluded that the large gauge gives a better estimate of the amount of rain falling on the field. Daily readings are available for the whole period, the rain up to 9 a.m. being ascribed to the previous day. The rainfall was divided as explained in Section 3 into 61 periods of 6 days each, and each set of 61 values was then analysed by calculating the coefficients of the polynomials up to the 5th degree. Thus the amount and distribution of rain in each season was represented by a series of 6 numbers; such a representation of a complicated sequence of events might seem to be insufficient, but it has been shown in Section 3, that only such coefficients are required as correspond to the regression function; since the latter varies relatively slowly, little would be gained by following in more detail the rapid fluctuations of the weather.

The computation of the rainfall coefficients involved a great deal of labour. The method

TABLE III.

—	2B.	3 & 4.	5.	6.	7.	8.	10.	11	12.	13.	14.	17 and 18.	
												Minerals.	Ammon. Sulphate.
1854	+ 9·54	+ 6·86	+ 5·97	+ 7·62	+11·95	+13·98	+12·79	+15·83	+13·16	+12·98	+12·44	+ 6·43	+13·23
	+ 0·45	+ 1·33	− 0·42	− 0·06	− 2·93	− 5·65	− 1·14	− 7·67	− 2·61	− 2·80	− 2·62	− 0·67	+ 1·04
1856	+ 0·15	− 1·59	+ 0·54	− 1·34	− 0·75	− 0·03	− 0·07	+ 0·91	− 2·44	− 3·45	− 0·99	− 1·66	− 2·00
	+ 3·81	+ 3·89	+ 4·63	+ 5·69	+ 6·03	+ 7·82	+ 5·25	+ 8·10	+ 6·90	+ 6·98	+ 6·74	+ 6·26	+ 7·02
1858	+ 0·47	+ 1·14	− 0·08	− 1·05	− 0·52	+ 0·51	− 1·53	+ 0·63	+ 0·20	+ 0·38	+ 0·93	+ 1·39	− 0·26
	− 2·45	+ 1·45	+ 1·73	+ 0·01	− 5·04	− 7·39	− 4·70	− 4·10	− 2·84	− 2·62	− 2·85	+ 0·08	− 1·21
1860	− 6·52	− 3·44	− 2·64	− 7·61	−11·87	−10·85	− 9·93	− 8·96	−10·19	−10·56	−10·08	− 4·46	− 8·53
	− 3·79	− 5·04	− 2·74	− 1·55	− 4·26	− 6·94	−12·12	− 6·74	− 4·32	− 2·89	− 3·52	− 1·20	− 0·60
1862	+ 0·26	− 0·06	+ 0·08	− 0·32	− 2·70	− 2·20	− 2·20	− 3·74	− 2·99	− 4·21	− 5·30	− 0·92	− 5·37
	+ 6·48	+ 2·86	+ 2·50	+11·80	+15·86	+14·56	+15·69	+15·67	+17·88	+17·10	+18·00	+ 2·37	+13·66
1864	+ 3·20	+ 1·41	+ 0·21	+ 4·15	+ 8·88	+ 9·38	+ 9·05	+ 6·98	+ 9·23	+ 7·63	+ 6·28	− 0·56	+ 4·10
	+ 1·12	− 0·79	− 1·94	− 1·20	+ 4·29	+ 3·84	+ 3·36	− 1·47	+ 0·07	+ 2·26	+ 2·12	− 0·54	− 0·09
1866	− 2·53	− 1·50	− 2·34	− 4·79	+ 5·01	− 6·82	+ 3·57	− 0·05	− 5·31	− 9·35	− 5·63	− 4·18	− 4·54
	− 6·82	− 4·66	− 5·82	− 8·67	−11·75	− 7·72	− 4·37	− 5·23	− 8·14	− 9·51	−10·00	− 5·39	− 6·79
1868	+ 8·25	+ 3·87	+ 3·01	+ 4·73	+ 6·92	+ 9·20	+ 3·78	+ 6·95	+ 8·13	+ 6·68	+ 9·78	+ 3·21	+ 7·58
	+ 5·51	+ 1·83	+ 1·61	− 1·26	− 3·58	− 1·75	− 2·08	− 3·63	− 3·58	− 4·49	− 3·40	+ 1·29	− 6·38
1870	+ 4·45	+ 3·11	+ 5·00	+ 8·26	+ 9·41	+ 9·48	+ 1·32	+ 0·13	+ 5·29	+ 5·93	+ 5·38	+ 4·78	+ 5·46
	+ 7·55	− 1·99	− 1·44	− 4·60	− 8·10	− 7·36	−10·48	−13·51	− 8·08	− 0·20	− 5·34	+ 2·27	− 0·02
1872	+ 1·43	− 0·44	− 0·20	− 0·55	+ 0·01	+ 1·13	− 1·59	+ 3·36	+ 0·87	+ 0·17	+ 1·38	− 0·24	− 2·18
	− 3·80	+ 0·78	− 0·01	− 4·74	− 7·18	− 6·40	+ 0·72	− 4·01	− 4·78	− 5·75	− 4·25	− 1·03	− 7·50
1874	+ 8·97	+ 0·96	+ 0·52	+ 4·50	+10·76	+ 7·04	+ 7·29	+10·09	+12·38	+ 8·38	+ 8·76	+ 1·54	+ 5·63
	− 1·24	− 2·59	− 3·09	− 3·74	− 2·48	− 3·21	− 4·85	− 4·38	− 1·31	− 1·16	− 1·12	− 0·47	− 1·89
1876	− 6·22	− 2·03	− 1·70	− 4·22	− 4·69	− 3·30	− 4·87	− 7·59	− 6·97	− 2·85	− 5·00	− 1·55	− 0·94
	− 6·05	− 1·17	− 0·49	− 5·34	− 8·20	− 8·04	− 0·12	− 3·71	− 8·11	− 9·54	− 8·27	− 1·86	−14·80
1878	− 2·13	+ 1·92	+ 2·55	+ 2·80	+ 3·15	+ 5·39	+10·99	+ 8·27	+ 3·66	+ 1·81	+ 5·45	+ 3·47	+ 1·54
	−14·69	− 5·95	− 6·53	− 9·55	−12·00	−12·25	−13·13	− 9·94	−11·34	−11·58	−10·29	− 8·65	− 7·26
1880	+ 7·27	+ 1·54	+ 5·43	+ 7·15	+ 5·97	+ 2·30	− 5·25	+ 4·84	+ 4·03	+ 5·34	+ 4·50	+ 3·12	+ 4·99
	− 1·35	+ 2·45	+ 0·30	+ 1·12	− 2·17	− 2·58	+ 1·76	+ 0·65	− 1·56	+ 0·48	+ 0·91	+ 1·20	+ 3·93
1882	+ 0·58	+ 0·09	− 0·07	+ 2·22	+ 6·54	+ 3·44	+ 7·69	+ 9·58	+ 9·28	+ 4·25	+ 8·10	+ 3·41	+ 2·69
	+ 2·44	+ 2·56	+ 2·96	+ 6·37	+ 6·36	+ 7·83	+ 0·71	+ 5·68	+ 5·24	+ 5·92	+ 6·37	+ 3·17	− 0·46
1884	− 1·00	+ 1·83	+ 2·52	+ 4·44	+ 8·16	+ 9·02	+ 8·80	+11·35	+10·08	+ 4·58	+ 9·26	+ 0·43	+ 4·80
	+ 5·90	+ 2·63	+ 2·01	− 0·13	+ 0·74	+ 1·76	+ 7·17	+ 1·64	+ 1·46	− 1·56	− 0·23	− 0·22	+ 3·57
1886	+ 1·53	− 2·44	− 2·15	+ 0·43	+ 3·84	+ 6·83	− 4·49	− 3·51	+ 0·28	+ 8·25	+ 3·68	− 0·02	+ 7·95
	− 0·98	+ 2·48	+ 0·85	+ 0·26	− 2·47	− 1·61	+ 4·24	+ 1·14	+ 3·59	− 3·16	+ 1·27	− 3·05	+ 0·48
1888	+ 1·54	− 1·91	− 1·93	+ 0·06	+ 3·32	− 1·36	− 5·87	− 9·59	− 3·14	+ 3·48	− 1·43	− 0·93	+ 1·33
	+ 3·33	+ 0·18	+ 0·99	− 0·96	− 2·46	− 1·77	− 5·71	− 4·84	− 2·95	− 4·28	− 3·26	− 4·11	− 7·69
1890	—	—	—	—	—	—	—	—	—	—	—	—	—
	—	—	—	—	—	—	—	—	—	—	—	—	—
1892	− 5·61	− 3·41	− 4·46	− 2·84	− 2·57	− 0·74	+ 7·03	− 5·42	− 4·00	− 2·56	− 4·31	− 2·95	− 3·03
	− 5·18	− 2·28	− 0·71	− 5·79	−14·55	−17·45	−10·31	−13·48	−16·97	−15·29	−15·64	− 3·18	−11·87
1894	+ 5·72	+ 4·80	+ 7·33	+12·71	+13·29	+ 9·41	+11·30	+17·87	+18·61	+15·72	+15·77	+11·57	+ 4·72
	+ 3·86	− 2·50	− 0·04	− 4·22	− 2·96	+ 0·09	− 8·76	− 5·19	− 5·57	− 3·49	− 6·41	− 3·26	− 3·52
1896	+ 3·86	+ 3·39	+ 5·04	+ 4·55	+ 2·03	+ 4·15	+ 3·30	+ 3·05	+ 5·06	− 4·58	+ 1·11	+ 1·10	+ 3·25
	− 2·88	− 3·74	− 2·34	− 5·81	− 6·49	− 3·20	− 2·12	− 4·84	− 7·13	− 4·72	− 8·55	− 4·98	− 1·95
1898	− 1·99	− 0·43	− 2·16	− 4·21	− 6·51	−10·83	+ 1·65	− 3·70	− 4·12	− 5·66	− 3·50	− 0·93	− 6·27
	+ 2·78	− 0·75	− 2·18	− 5·89	− 3·28	− 0·95	+ 3·81	+ 0·92	− 0·10	− 4·27	+ 0·88	− 2·01	− 5·29
1900	− 6·06	− 0·28	− 0·04	− 4·96	− 4·30	+ 4·16	+ 0·77	− 1·77	− 4·00	− 2·72	− 3·66	− 3·78	− 2·75
	+ 0·85	− 0·51	− 0·21	− 1·44	− 3·95	+ 2·88	+ 1·76	+ 0·69	− 1·77	− 2·47	− 3·87	+ 2·40	− 2·86
1902	+ 3·41	+ 1·36	+ 1·05	+ 3·15	+ 5·50	+ 6·17	+ 5·00	+ 3·70	+ 5·20	+ 7·70	+ 6·67	+ 5·12	+ 5·85
	− 7·60	− 4·09	− 5·47	− 4·65	− 5·29	− 2·66	− 4·98	− 3·23	−10·39	− 4·71	− 8·08	− 8·75	− 4·70
1904	−14·09	− 7·00	− 6·83	−10·16	−11·51	−12·87	− 7·81	− 5·98	−11·42	− 9·67	− 7·14	−10·67	− 8·99
	—	—	—	—	—	—	—	—	—	—	—	—	—
1906	—	—	—	—	—	—	—	—	—	—	—	—	—
	+ 2·91	− 0·82	− 1·17	+ 4·78	+ 5·77	− 0·34	+10·45	+15·10	+16·01	+ 5·98	+14·58	− 1·98	+ 3·59
1908	+ 7·76	+ 2·90	+ 3·96	+ 3·76	+ 6·53	+13·54	+ 5·04	+ 4·51	+ 7·95	+ 8·00	+ 4·93	+ 2·47	+ 6·73
	+ 2·90	− 0·01	− 1·48	− 0·02	+ 3·21	− 0·56	− 5·89	− 9·89	− 4·63	+ 0·62	− 4·36	− 2·77	+ 4·39
1910	− 0·70	− 1·26	− 1·46	+ 0·93	+ 0·87	− 4·02	− 1·71	+ 4·21	+ 2·80	− 1·01	+ 3·97	− 1·41	+ 1·76
	+ 7·92	+ 3·92	+ 3·65	+ 1·16	+ 1·76	+ 5·81	+ 7·31	+ 4·67	+ 4·17	+ 4·16	+ 4·86	+ 2·82	+ 3·61
1912	− 9·77	− 3·83	− 5·25	−13·03	−16·12	−18·99	−12·68	−12·33	−17·11	−18·62	−15·95	− 4·73	−15·91
	− 5·02	− 3·10	− 3·64	− 3·89	− 6·30	− 3·26	− 5·15	− 3·77	− 4·00	− 5·79	− 2·95	− 3·37	− 2·76
1914	+ 3·65	− 2·51	− 0·62	+ 1·91	+ 0·96	+ 0·23	− 2·40	− 0·49	+ 1·50	− 4·16	− 3·85	− 5·11	− 5·19
	—	—	—	—	—	—	—	—	—	—	—	—	—
1916	− 6·70	− 3·70	− 2·98	− 4·60	− 2·22	−10·26	+ 4·11	− 0·32	− 3·55	− 2·92	− 3·81	− 5·22	− 0·94
	−10·99	− 1·48	− 1·31	+ 2·50	+ 0·17	+ 4·03	− 1·60	− 3·42	− 3·16	+ 4·39	− 1·84	+ 2·82	+ 2·12
1918	+ 6·60	− 0·03	− 2·11	− 3·19	− 1·48	+ 2·05	− 0·68	− 0·93	+ 1·32	− 0·74	+ 1·52	+ 2·11	+ 0·39

employed was that of successive summation, introduced by G. F. Hardy. By this method from a column of numbers $x_1, x_2, \ldots, x_n$ we obtain in succession

$$S_1 = x_1 + x_2 + x_3 + \ldots + x_n = S(x)$$

$$S_2 = nx_1 + (n-1)x_2 \ldots + 2x_{n-1} + x_n = S\{(n+1-r)x_r\}$$

$$S_3 = \frac{n(n+1)}{2}x_1 + \frac{(n-1)n}{2}x_2 + \ldots + 3x_{n-1} + x_n = S\left\{\frac{(n+1-r)(n+2-r)}{1.2}x_r\right\},$$

and so on. The successive summations carried as far as S_6 lead to very large numbers, but the work is straightforward and easily checked. It is not necessary to carry out the summation of the column in one piece. It may with advantage be broken in the middle; for example in the majority of the rainfall analyses the first 29 terms were summed as above, giving

$$z_1 = \overset{29}{\underset{1}{S}}(x_r)$$

$$z_2 = \overset{29}{\underset{1}{S}}\{(30-r)x_r\}$$

$$z_3 = \overset{29}{\underset{1}{S}}\left\{\frac{(30-r)(31-r)}{1.2}x_r\right\} \text{ and so on;}$$

while the remaining 32 values were summed backwards from the bottom of the column, dropping one term at the end of each summation, and so giving

$$z_1' = \overset{61}{\underset{30}{S}}(x_r)$$

$$z_2' = \overset{61}{\underset{31}{S}}\{(r-30)x_r\} = -\overset{61}{\underset{30}{S}}\{(30-r)x_r\}$$

$$z_3' = \overset{61}{\underset{32}{S}}\left\{\frac{(r-30)(r-31)}{1.2}x_r\right\} = \overset{61}{\underset{30}{S}}\left\{\frac{(30-r)(31-r)}{1.2}x_r\right\} \text{ and so on.}$$

Taking now alternately sums and differences, we obtain

$$S_1' = z_1 + z_1' = \overset{61}{\underset{1}{S}}(x_r)$$

$$S_2' = z_2 - z_2' = \overset{61}{\underset{1}{S}}\{(30-r)x_r\}$$

$$S_3' = z_3 + z_3' = \overset{61}{\underset{1}{S}}\left\{\frac{(30-r)(31-r)}{1.2}x_r\right\} \text{ and so on;}$$

from which the final sums of the whole column may be obtained from the equations

$$S_1 = S_1'$$

$$S_2 = S_2' + 32S_1'$$

$$S_3 = S_3' + 32S_2' + \frac{32.33}{1.2}S_1'.$$

The series $S_1, S_2, S_3, \ldots\ldots$, by whichever method it is obtained, is divided by the series of divisors

$$61, \quad \frac{61 \cdot 62}{1 \cdot 2}, \quad \frac{61 \cdot 62 \cdot 63}{1 \cdot 2 \cdot 3} \quad \text{and so on,}$$

yielding a series of numbers of similar magnitude

$$a = \frac{1}{61} \mathrm{S}(x_r)$$

$$b = \frac{1 \cdot 2}{61 \cdot 62} \mathrm{S}\{(n+1-r)\, x_r\}$$

$$c = \frac{1 \cdot 2 \cdot 3}{61 \cdot 62 \cdot 63} \mathrm{S}\left\{\frac{(n+1-r)\,(n+2-r)}{1 \cdot 2}\, x_r\right\}$$

with similar equations for d, e, and f. These may be regarded as weighted means of the series $x_1, \ldots, x_n$; it will be observed that the weights involve r, and therefore the time, up to the 5th degree. The series is therefore equivalent for the purpose of evaluating the polynomial coefficients to the first five moments of rainfall as distributed in time. The values at which we have arrived are, however, more convenient for computing purposes than the moments would be, for the quantities required by the method of Section 3 are

$$\rho_0 = \mathrm{S}(x\mathrm{T}_0) = \frac{1}{\sqrt{n}} \mathrm{S}(x) = \sqrt{n}\, a$$

$$\rho_1 = \mathrm{S}(x\mathrm{T}_1) = \sqrt{\frac{12}{n(n^2-1)}}\, \mathrm{S}(tx) = \sqrt{\frac{3n(n+1)}{n-1}}\, (a-b)$$

$$\rho_2 = \mathrm{S}(x\mathrm{T}_2) = \sqrt{\frac{180}{n(n^2-1)(n^2-4)}}\, \mathrm{S}\left\{\left(t^2 - \frac{n^2-1}{12}\right) x\right\}$$

$$= \sqrt{\frac{5n(n+1)(n+2)}{(n-1)(n-2)}}\, (a - 3b + 2c) \text{ and so on.}$$

The factors under the square root are not necessary for the correlational work, so that actually it is only necessary to calculate for each year

$$\left.\begin{aligned} a' &= a \\ b' &= a - b \\ c' &= a - 3b + 2c \\ d' &= a - 6b + 10c - 5d \\ e' &= a - 10b + 30c - 35d + 14e \\ f' &= a - 15b + 70c - 140d + 126e - 42f \end{aligned}\right\} \quad \ldots\ldots \quad \text{(V)}$$

These are the independent variates in terms of which the wheat yield is to be expressed

in the form of a regression equation. The regression coefficients of the yield upon these will have to be divided by factors of the form

$$\sqrt{\frac{(2s+1)\,.\,n(n+1)\ldots(n+s)}{(n-1)\ldots(n-s)}}$$

in order to give the coefficients α_s of Section 3, but as in the expansion of the regression function, α_s is multiplied by

$$\sqrt{\frac{(2s+1)\,.\,((2s)!)^2}{(s!)^4\,n(n^2-1)\ldots(n^2-s^2)}}$$

we actually multiply the regression coefficients by

$$\frac{(2s)!}{(s!)^2\,n(n+1)\ldots(n+s)}$$

in order to obtain the coefficients of the polynomial,

$$t^s + \ldots,$$

in the expansion of the regression function.

6. *The Rain Distribution Values.*

The values actually obtained for the quantities a', ..., f', for the 65 periods ending August 31st, 1854 to 1918, are given in Table IV; for tabulation they have been multiplied by 1,000; thus the value 347 in the second column shows that the average rainfall for the first period was 347 thousandths of an inch every 6 days. The figures in the third column measure the average rate at which rainfall was increasing or decreasing during the period, as indicated by a straight line fitted to the recorded values; c' measures the parabolic term in the rainfall sequence, and so on taking more and more complex features of the distribution into account.

These values are themselves of very great interest, since the incidence of rainfall has not previously been analysed in this way.

The individual peculiarities of the successive seasons are brought out clearly, and it is possible to examine the sequence of years by adequate statistical methods. We may first enquire whether the observed sequence accords with the view that each season is an independent product of random causes under constant climatic conditions, or whether on the other hand the sequence indicates progressive changes in the quantity and distribution of the rainfall.

The series of 65 values of a' was therefore analysed by summation in the same manner as the 61 values of the rainfall record of each year had been treated; as an illustration

TABLE IV.

Harvest Year.	a'	b'	c'	d'	e'	f'	Harvest Year.	a'	b'	c'	d'	e'	f'
1854	347	− 14	+ 33	− 16	− 21	− 1	1887	387	− 82	− 6	+ 29	− 18	+14
1855	398	+ 90	+ 41	+ 8	− 35	− 22	1888	500	+ 48	+ 59	− 20	− 10	−13
1856	487	− 18	+ 12	+ 1	− 27	+ 35	1889	498	+ 72	− 1	− 11	− 36	−10
1857	404	− 9	+ 19	+ 15	− 9	− 1	1890	450	+ 20	+ 35	− 1	− 15	−12
1858	441	− 69	+ 69	− 38	− 12	+ 15	1891	383	+ 65	+ 35	+ 22	0	+28
1859	412	+ 64	+ 3	− 14	− 4	− 5	1892	487	− 29	+ 70	+ 45	− 35	+29
1860	598	+ 39	+ 35	− 12	− 12	+ 11	1893	398	− 38	+ 37	+ 26	− 16	− 4
1861	381	− 8	+ 3	− 17	− 24	+ 1	1894	488	− 7	+ 24	+ 30	− 8	+16
1862	451	+ 22	− 13	− 13	− 21	+ 18	1895	474	− 3	+ 66	+ 51	− 42	−11
1863	414	− 21	+ 36	+ 19	− 7	+ 14	1896	399	− 27	+ 21	+ 32	− 4	+13
1864	331	− 56	+ 16	− 24	+ 3	+ 3	1897	618	−131	+ 77	− 32	+ 55	− 5
1865	454	+ 43	+ 61	− 30	+ 29	− 11	1898	323	− 11	+ 9	− 20	+ 4	−17
1866	568	− 2	+ 3	+ 38	− 45	+ 8	1899	405	− 20	− 42	+ 27	− 17	+ 5
1867	513	− 14	+ 21	− 27	+ 7	− 17	1900	514	− 47	+ 27	+ 35	+ 4	+ 2
1868	346	− 11	+ 14	+ 26	+ 42	− 11	1901	406	− 10	− 7	+ 31	+ 2	+11
1869	426	− 44	− 37	+ 10	− 3	− 3	1902	382	+ 30	+ 18	+ 12	− 5	+ 2
1870	354	− 65	+ 10	+ 21	+ 9	− 11	1903	520	+106	+ 28	− 12	− 8	−16
1871	451	− 7	+ 16	− 23	− 32	− 16	1904	517	− 75	+ 30	+ 32	+ 2	+ 7
1872	475	+ 8	+ 13	− 23	+ 5	− 6	1905	428	+ 35	+ 58	0	+ 17	+11
1873	503	− 57	+ 6	+ 64	− 31	+ 2	1906	389	− 37	− 13	+ 11	− 9	−15
1874	357	− 12	+ 30	− 10	− 3	− 6	1907	481	− 44	+ 13	+ 5	− 47	+24
1875	526	− 11	+ 42	− 33	− 26	− 42	1908	494	− 5	+ 6	+ 24	− 2	+38
1876	524	− 80	+ 29	+ 18	− 8	+ 42	1909	419	+ 52	+ 26	− 26	− 5	− 7
1877	648	− 76	+ 9	+ 22	+ 23	− 38	1910	507	− 16	+ 16	+ 23	− 11	+ 5
1878	532	+ 38	+ 10	+ 20	+ 1	+ 35	1911	465	− 43	− 38	+ 25	− 35	+11
1879	674	+105	+ 37	+ 26	− 35	− 3	1912	672	+ 18	+ 33	+109	− 9	− 2
1880	350	+ 39	+ 25	− 29	− 7	− 24	1913	451	− 54	− 35	+ 20	− 2	+18
1881	603	− 78	+110	+ 35	+ 41	+ 16	1914	419	− 42	− 14	− 10	+ 8	−57
1882	530	− 11	− 8	+ 5	− 28	+ 15	1915	604	− 14	− 36	+ 65	− 29	−55
1883	573	− 81	+ 6	+ 25	− 39	− 14	1916	582	− 17	− 26	+ 51	+ 15	+14
1884	422	− 53	+ 35	+ 1	0	+ 8	1917	577	+ 54	+ 82	+ 58	− 25	− 2
1885	434	− 20	− 37	− 11	+ 9	0	1918	449	− 18	+ 15	− 22	− 11	+23
1886	508	− 83	+ 33	− 30	− 13	0							

of the method the numerical values are given in Tables V and VI; before summation each a' was reduced by 400, the true value of the mean being inserted in the fourth column. The second column gives the successive sums, S_1 to S_6; from these the third column is derived by dividing by factors of the form

$$\frac{65 \,.\, 66 \ldots (65 + r)}{r!}, \qquad r = 1, \ldots, 5.$$

Thus the third column gives the values of a to f for the series a'. The fourth column gives the corresponding values a' to f' for the series a', obtained by equations (V); while the fifth column, obtained by multiplying by factors of the form

$$\sqrt{\frac{(2r+1) \,.\, 65 \ldots (65 + r)}{64 \ldots (65 - r)}}.$$

are the actual values of the first five transformed co-ordinates spoken of in Section 4, as x_2' to x_6'.

TABLE V.—Analysis of sequence of values of a'.

—	S_1 to S_6.	a to f.	a' to f'.	x_2' to x_6'.
1	4,521	69·55385	469·55385	
2	128,341	59·832634	+9·72122	+137·85
3	2,603,055	54·3378562	−1·26834	− 23·95
4	40,090,879	49·22841040	+7·79456	+182·35
5	500,901,859	44·570118750	+2·35049	+ 66·31
6	5,313,933,165	40·528492120	−2·62489	− 88·43

In an unchanging series the values x_2', x_3' ... vary about zero in an approximately normal distribution, the standard deviation of which may be obtained from that of the original series; for

$$\overset{n}{\underset{2}{S}}(x_r'^2)$$

is the sum of the squares of the deviations of the original series from their mean. Slow changes in the original series will be indicated by high positive or negative values in x_2', x_3', ..., and if such slow changes are suspected, it will be better to estimate the variance due to random causes from

$$\overset{n}{\underset{7}{S}}(x_r'^2),$$

from which the first five values have been omitted. From the sums of the squares of the deviations we may thus obtain a series of values each obtained from the last by deducting the square of the corresponding value x_r'; from each such sum may be obtained an estimate of the standard deviation due to random causes, by dividing by the number of squares concerned (degrees of freedom), and taking the square root. Such estimates will be equivalent to those derived from the residuals left after polynomials of the first to the fifth degree have been successively fitted; but the labour of calculating the polynominal values is avoided.

TABLE VI.

Degrees of freedom.	Sum of squares.	Mean square.	Standard deviation.
64	465,105	7267·3	85·25
63	446,103	7081·0	84·15
62	445,529	7186·0	84·77
61	412,278	6758·7	82·21
60	407,881	6798·0	82·45
59	400,061	6780·7	82·35

The diminution of the estimates of the standard deviation indicates that the first 5 values of x' are on the whole higher than those which follow; for example x_4' is more than double the standard, and suggests strongly that real changes are taking place in the rainfall. To test this more accurately, divide the sum of the squares of the five deviations by the mean square of the remainder, then

$$\chi^2 = \frac{465105 - 400061}{6780{\cdot}7} = 9{\cdot}59,$$

whence, entering ELDERTON's table with $n' = 6$, we obtain $P = {\cdot}089$. Thus a larger value of χ^2 would be obtained by chance only 8·9 times in a hundred, from a series of values in random order. There is thus some reason to suspect that the distribution of rainfall in successive years is not wholly fortuitous, but that some slowly changing cause is liable to affect in the same direction the rainfall of a number of consecutive years. Another way of putting the same result is that the variance estimated from the residuals of a polynomial of the 5th degree is 93·3 per cent. of the variance of the original series, so that some 6·7 per cent. of the total variance observed in annual rainfall may be ascribed to slow changes, while the remaining 93·3 per cent. of the variance are due to the chance circumstances of each particular year.

We summarise below the results of applying a similar analysis to the rainfall distribution values a' to f' :—

TABLE VII.

——	a'	b'	c'	d'	e'	f'
Mean	469·55	− 11·09	+ 19·22	+ 9·43	− 8·69	+ 0·57
x_2'	+137·85	− 18·62	− 35·01	+ 86·92	+ 8·78	− 2·11
x_3'	− 23·95	+ 56·19	− 33·75	+ 5·45	− 16·94	− 0·84
x_4'	+182·35	− 35·31	− 33·96	+ 13·02	+ 15·72	− 23·80
x_5'	+ 66·31	+ 13·00	+ 45·52	+ 5·76	− 29·78	+ 1·25
x_6'	− 88·43	+ 24·69	+ 33·79	− 20·47	+ 11·50	+ 22·33
Standard Residue	82·35	50·43	30·11	28·19	20·76	19·91
χ^2	9·59	2·17	7·42	10·33	3·78	2·70
P	0·089	0·83	0·19	0·067	0·58	0·74

It is quite clear, from the values of P, that no significant changes are observable in b', c', e' and f'. These fluctuate with large standard deviations about the mean values

given in the first line of Table VII, which mean values have not significantly changed during the 65 years examined. With a', and more clearly with d', the mean appears to show gradual changes. In the latter case the change observable is very clear, and of a simple character. The value of x_2', representing a linear increase, exceeds the standard deviation in the ratio 3·083; the probability of such a value occurring by chance is only 0·00205. The remaining values, x_3' to x_6', are individually and collectively insignificant, whence it appears that the slow change in d' is significant, and may be represented by a uniform increase. This is in sharp contrast to the behaviour of a', a contrast which is brought out by plotting successive 10 year means of these two quantities.

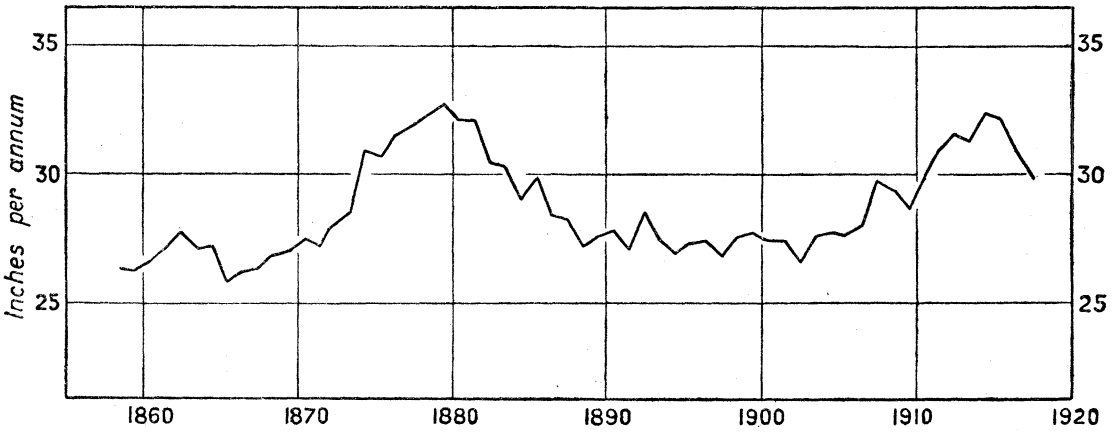
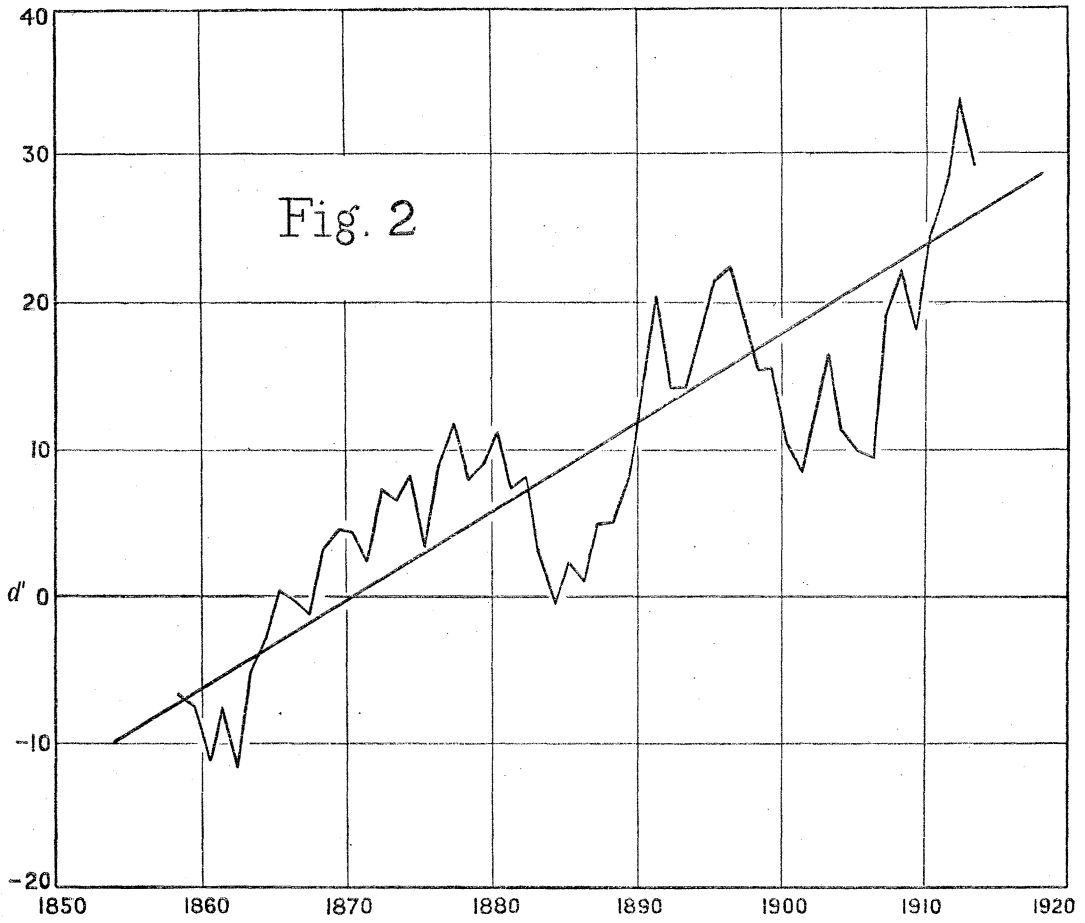


Fig. 1.

In the case of the total rainfall (fig. 1) measured by a', there would seem to have been an excess of rain for a series of years about 1879, and a second spell of about equal intensity about 1914; the interval being 35 years. At these periods the annual rainfall average rises to about 32 inches. Prior to the first wet spell, and in the intermediate period, the average annual rainfall is about 27 inches. Such a change is small compared to the annual fluctuations; we have seen that it accounts for only about 6·7 per cent. of the variance, and that the whole effect is scarcely significant in our data. The distribution value d', on the other hand, which measures not the total rainfall but its distribution over the year, shows a distinct and apparently uniform increase. Judging from the straight line (fig. 2), its mean value has changed from -10 in 1854, to nearly $+29$ in 1918; its mean value $+9·43$, given above, is thus not a permanent feature of the climate of the district. As in all rainfall features the annual fluctuation is very great; of the variation observed in d' in the 65 years only 12·3 per cent. is ascribable to the linear change, the remaining 87·7 per cent. being apparently due to random fluctuations. In the absence of a similar analysis of rainfall at other stations, it would be premature to discuss the possible causes of this remarkable and progressive change in the climate. It may be remarked that such little additional information as is to be obtained from the monthly records indicates that the most marked feature of the change in progress is an



Ten year means of the distribution value d'.

increase in the December rainfall with perhaps some relative reduction of rain in Spring and Autumn.

The mean values of c', d', and e', differ significantly from zero, owing to the seasonal variations of rainfall. The series of mean values, neglecting the mean value of f' which is altogether negligible, represent the mean course of the rainfall sequence during the year. This is represented in fig. 3, where the mean monthly values have been inserted for comparison; these values have been reckoned *per day* to eliminate the effect of the varying lengths of the months. The monthly means are of course subject to large probable errors, and the smooth curve gives a slightly better estimate of the average sequence; this sequence is necessarily in process of modification owing to the progressive increase of d'.

The standard deviations of the six distribution values are in good mutual agreement and evidently arise from a common cause, namely the random fluctuation of rain in a six-day period. This may be seen by multiplying them by the factors necessary to transform a', ..., f' into $\rho_1, \rho_2, \ldots, \rho_6$ (Section 5), that is by

$$\sqrt{61},\ \sqrt{\frac{3.61.62}{60}},\ \text{etc.};$$

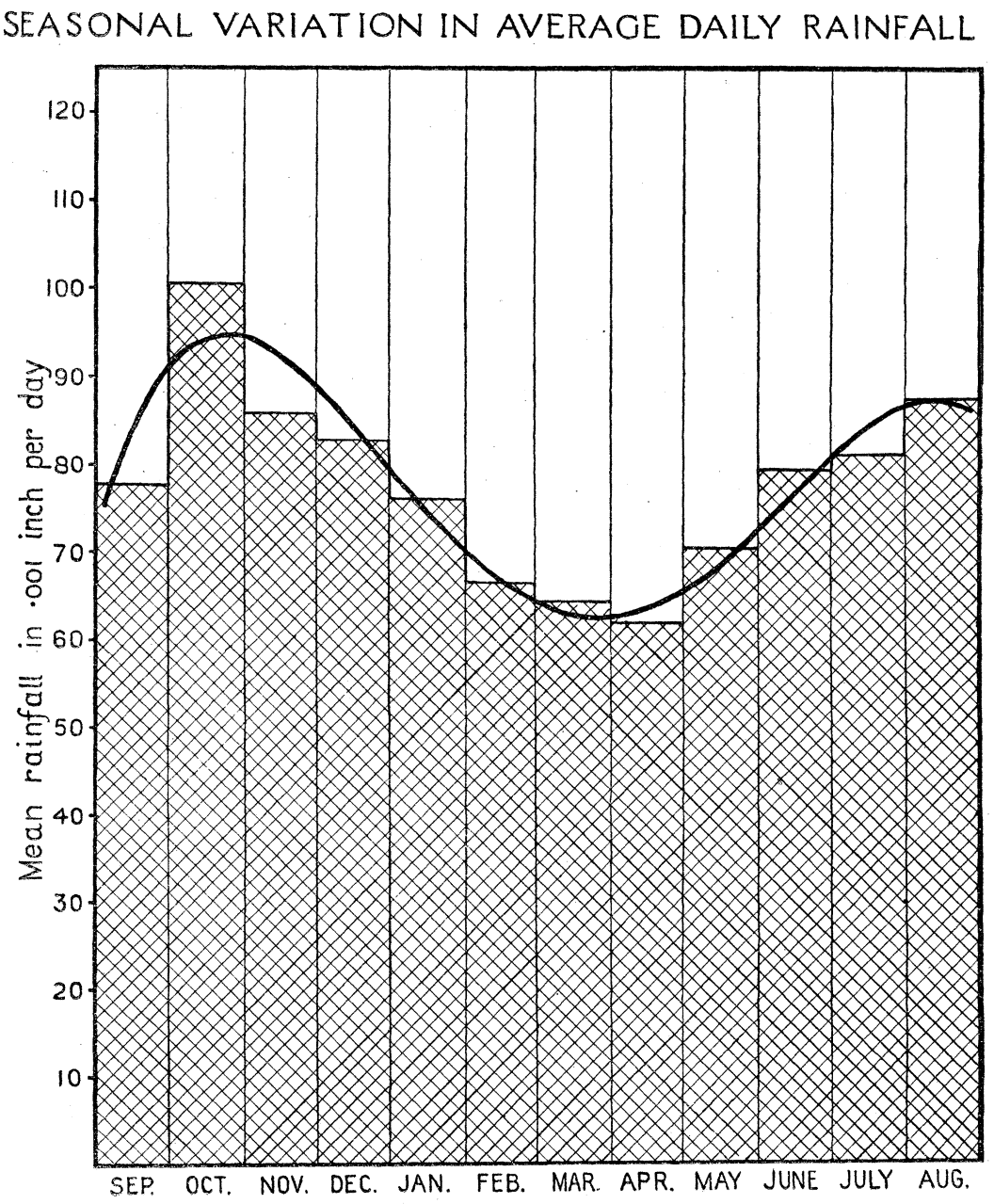


Fig. 3.

the standard errors of $\rho_1, \ldots, \rho_6$ are then

σ_{ρ_1}.	σ_{ρ_2}.	σ_{ρ_3}.	σ_{ρ_4}.	σ_{ρ_5}.	σ_{ρ_6}.	Mean.
643	693	552	643	573	660	627

These may be regarded as independent estimates of the standard deviation of the rainfall (in thousandths of an inch) in a six-day period, each derived from 59 independent squares, which is equivalent to a random sample of 60 values. The variation of these estimates is just of the order to be expected on the basis of random sampling. The

variation of the rainfall distribution values is thus nearly what we should deduce from the supposition that in addition to the small contributions made by secular and seasonal variation, each 6 day period varied independently with a standard deviation 0·627 inch, or a coefficient of variation of 134 per cent.

The distribution of the distribution values are not materially skew as far as can be judged from a series of 65 values. The following values obtained from the moments of the sample show the observed deviations from normality, with the standard errors appropriate to a normal population

	a'.	b'.	c'.	d'.	e'.	f'.	Standard error.
$\gamma_1 = \sqrt{\beta_1}$	+ 0·493	+ 0·336	+ 0·232	+ 0·566	+ 0·614	− 0·546	± 0·306
$\gamma_2 = \beta_2 - 3$	− 0·290	+ 0·066	+ 0·354	+ 0·510	+ 0·920	+ 1·006	± 0·612

These figures suffice to show that the observed values hardly differ significantly from samples of normal distributions, although there are indications that, if the values for a considerably longer series of years were available, real departures from the normal distribution would become apparent.

7. *Correlations of Rainfall Distribution Values.*

The rainfall distribution values have been calculated from uncorrelated functions of the time, and if the rainfall at different periods of the year were uncorrelated the rainfall distribution values should also be uncorrelated. If, on the other hand, rainfall at one time of the year were associated with rainfall or lack of rainfall at another time, then correlations would appear in the rainfall distribution values, and such correlations would indicate the nature and extent of such mutual correlations between different parts of the season. Hooker observed, in the short period of 21 years at his disposal, a correlation as high as + 0·6 between the winter rainfall (1st to 8th weeks) and that of the previous autumn (37th to 44th weeks), and concluded that so large a correlation indicated a real interdependence between winter and autumn rainfall. More recently W. T. Russell (13, 1922) has calculated the correlation coefficients for monthly rainfall between consecutive and between alternate months. For Greenwich (65 years) no appreciable correlations are found between successive months, but in the alternate months it is found that June and August have a positive correlation + 0·55. Such a value should occur by chance from uncorrelated material once only in 900,000 times, so that even allowing for the fact that it is the highest of the 24 values obtained, it would appear to be significant of a real association. It should, however, be borne in mind that any secular changes either in total rainfall, or in its distribution through the year, as may exist at Greenwich, have not been eliminated in this coefficient, and, in the second place, the distribution of monthly rainfall is far from normal, and in consequence the probable error of such a determination may be distinctly higher than that calculated for normal distributions.

The correlations obtained between the 65 values $a', \ldots, f'$ are not, in any case, high. The transformed values ($z = \tanh^{-1} r$) are set out in Table VIII, since these values are distributed in random samples in approximately normal distributions, with the same standard error $\pm 0{\cdot}127$.

TABLE VIII.—Values of $z = \tanh^{-1} r$.

——	a'.	b'.	c'.	d'.	e'.
b'	−0·01	—	—	—	—
c'	+0·19	+0·09	—	—	—
d'	+0·38	−0·07	−0·09	—	—
e'	−0·06	−0·23	+0·18	−0·16	—
f'	−0·03	−0·08	+0·08	+0·16	−0·06

None of these correlations excite remark except that between a' and d'; these two variates are the two which show secular variation, and it is therefore of more interest to obtain the corresponding values after the secular variation has been removed. These are found by treating the sum of the products of deviations as the sum of squares has been treated in Table VI, deducting successively the products of the values $x_2', \ldots, x_6'$, for the two variates concerned, calculating r from the sums of squares similarly treated, and transforming to the z scale as before. The values so obtained are shown in Table IX, with standard error $\pm 0{\cdot}1325$.

TABLE IX.—Values of z after Eliminating Secular Change.

——	a'.	b'.	c'.	d'.	e'.
b'	+0·0342	—	—	—	—
c'	+0·2880	+0·0800	—	—	—
d'	+0·3103	−0·0526	−0·0228	—	—
e'	−0·0784	−0·2100	+0·2380	−0·1955	—
f'	+0·0356	−0·1119	+0·0381	+0·2092	−0·0525

The correlation of a' with d' now exceeds its standard error in the ratio 2·34; the probability of obtaining so large a value by chance from uncorrelated material is 0·019, and since this is the largest value of 15, it can scarcely be regarded as proof of association. To test the values collectively from the series of their squares, we obtain

$$\chi^2 = 22{\cdot}63, \qquad n' = 16, \qquad \mathrm{P} = 0{\cdot}093,$$

from ELDERTON's Table, showing that although there are signs of association among the rainfall distribution values, such association, if it exist, is not strong enough to show up significantly in a series of about 60 values.

8. *The Regression of Yield on the Distribution Values, and on the Rainfall at Different Seasons.*

The sums of squares and products of the distribution values provide a basis for the calculation of the partial regression coefficients of the yields upon them, but before applying them in this way it is necessary to allow for the fact that 5 years' yields, namely, 1890, 1891, 1905, 1906 and 1915, have been omitted; we must therefore deduct from the sums the contributions of these years. Table X shows the polynomial values of a', b', ... , f' for these years :—

TABLE X.

Year.	t.	a'.	b'.	c'.	d'.	e'.	f'.
1890	4	465·64	—13·21	33·17	9·11	— 9·73	4·16
1891	5	461·66	—12·36	33·36	9·10	— 9·42	4·84
1905	19	458·63	—11·96	10·57	18·88	— 6·03	2·00
1906	20	463·62	—12·52	7·98	20·29	— 6·17	0·97
1915	29	524·83	— 8·98	2·73	30·09	—10·66	— 4·13

calculated from the polynomial coefficients of Table VII. These values were subtracted from the observed values of Table IV, and the squares and products of the deviations deducted from the corresponding sums. The corrected sums, answering now to 54 degrees of freedom, were as follows :—

TABLE XI.

—	a'.	b'.	c'.	d'.	e'.	f'.
a'	+380,853	+ 14,984	+44,122	+38,013	— 5,278	+ 8,121
b'	+ 14,984	+140,110	+ 4,031	— 4,241	—14,577	— 8,921
c'	+ 44,122	+ 4,031	+49,302	+ 906	+ 6,746	— 1,392
d'	+ 38,013	— 4,241	+ 906	+44,944	— 5,790	+ 8,163
e'	— 5,278	— 14,577	+ 6,746	— 5,790	+24,429	— 2,769
f'	+ 8,121	— 8,921	— 1,392	+ 8,163	— 2,769	+19,670

Since we require to find the partial regressions for 13 separate plots, it is worth while to invert the determinant formed of these numbers, so as to obtain a matrix of multipliers each of which is the co-factor of the corresponding number above, divided by the determinant. Table XII shows these multipliers in millionths.

TABLE XII.

—	a'.	b'.	c'.	d'.	e'.	f'.
a	+3·235054	—0·304445	— 2·947149	— 2·535496	+ 0·672526	— 0·542793
b	—0·304445	+8·094433	— 1·082929	+ 1·042372	+ 5·775040	+ 4·100483
c	—2·947149	—1·082929	+24·070588	+ 0·730487	— 7·637592	+ 1·050675
d	—2·535496	+1·042372	+ 0·730487	+26·747433	+ 5·214984	— 8·794578
e	+0·672526	+5·775040	— 7·637592	+ 5·214984	+48·604978	+ 6·478923
f	—0·542793	+4·100483	+ 1·050675	— 8·794578	+ 6·478923	+57·557904

For each plot six correlation tables were constructed with the six rainfall distributions values, using the values in Tables III and IV, corresponding to the 60 years available. The six sums of products obtained from these tables, multiplied by the values of any column of Table XII and added, give the regression of yield on the corresponding rain variate. Table XIII gives the values obtained for these regression coefficients, the crop being measured in bushels per acre.

TABLE XIII.

Plot.	Regression on—					
	a'.	b'.	c'.	d'.	e'.	f'.
2B	− 40·9895	+ 2·1969	− 8·0261	− 46·2368	− 15·8457	+ 24·0548
3+4	− 20·2933	− 0·0582	+ 0·1506	+ 13·7263	+ 14·2433	+ 27·0821
5	− 20·7766	− 3·1878	− 0·0575	+ 13·9806	+ 7·2414	+ 18·8043
6	− 39·7077	− 0·6177	+ 34·1075	− 5·7556	− 8·3922	+ 51·2116
7	− 42·4430	− 3·1822	+ 36·3066	− 31·9965	− 16·6956	+ 83·5923
8	− 43·8350	− 11·7970	+ 35·9850	− 31·6292	+ 8·1368	+110·1433
10	− 25·7070	− 20·2730	− 15·3172	− 15·6733	+ 35·8060	+140·2458
11	− 30·4546	− 14·8637	− 9·8165	+ 1·6230	+ 17·7579	+113·9780
12	− 55·9573	− 13·9676	+ 20·9149	− 22·4509	− 23·0060	+113·0384
13	− 45·1447	− 3·9169	+ 63·5960	− 48·0067	− 24·4347	+ 92·2415
14	− 42·6026	− 12·9980	+ 25·9731	− 30·7156	− 24·8636	+115·5578
17 & 18 miner'ls	− 24·0377	+ 0·5955	+ 17·2943	+ 5·6818	− 3·8642	+ 33·4268
17 & 18 am'onia	− 42·6939	+ 3·8539	+ 60·9060	− 44·3881	+ 2·7304	+120·5257

These coefficients give directly linear regression formulæ expressing the deviation of the wheat crop on each plot in terms of the rainfall distribution values. Equally, as explained in Section 3, they enable us to estimate the average benefit or loss in bushels per acre ascribable to an additional inch of rain at any time during the year. For this purpose we divide the six regressions corresponding to any plot by

$$\frac{(r\,!)^2}{(2r)\,!}\,.\,61\,.\,62\,\ldots\,(61+r),$$

and we have the six coefficients of t', t, t^2 ... in

$$a = \mathrm{A} + \mathrm{B}t + \mathrm{C}\,(t^2 - n_2) + \ldots,$$

expressing a in terms of the polynomials of Section 3, wherein t is the time in 6-day intervals measured from the central period of the year. Figs. 4 to 8 show the course of the function a throughout the year.

9. *Discussion of Figs.* 4 *to* 8.

It should be emphasised that the information provided by a comparison of the rain record with the subsequent yields tells us the effect, not of so much rain, as such, but of

the total meteorological phenomena in fact associated with rain, at the time of the year considered. Thus in our records rain is associated with lower temperatures in summer, in winter with higher temperatures; and generally with diminished sunshine. The effects of these, to the extent in which they are associated, will be incorporated in the total effects shown in the charts, and are in fact an integral part of the value of a rain record as a means of foreseeing the prospects of the crop.

Even in the case of the rainfall itself, however, a detailed consideration of the ways in which it may affect the crop would lead to a most intricate discussion. It would be of the greatest value to know how important to the final crop, and how frequently influential, are not only the actual moisture available in the soil, but also the degree of soil aeration; how frequently, and to what extent, root development is hindered by soil saturation, or by an accumulation in toxic concentration of carbon dioxide. We should expect these factors to be intimately connected with rainfall, both in its direct effect in supplying fully aerated water, and in its indirect effects upon the soil texture.

In a single curve showing the average effect of rainfall (and of the average weather associated with such rainfall) upon the crop ultimately produced, all such contributory causes are included; by the comparison of the curves obtained from different plots, representing different manurial conditions of the soil, we may infer the effect of rainfall upon the availability of the manurial constituents of the soil. As will appear more fully, the predominant feature of such a comparison is the influence of excessive rain in removing soluble nitrates; this effect masks and overshadows all others, partly, I would suggest, by reason of its intrinsic importance to the wheat crop in our wet climate, partly because the plots on Broadbalk show great extremes in the relative abundance of available nitrogen.

The predominance in these curves of the effects which appear to be ascribable simply to the removal of soluble nitrate may perhaps explain two features which otherwise might be unexpected. In the first place the greater part of the effect of rainfall is expressible by means of the linear relations in the manure here represented; the quadratic terms, though of great interest and well worth further study, are of much less quantitative importance. Considering merely the effects of rain on the general environment of the growing plant we might expect definite optimum conditions throughout the year to be well marked, with correspondingly important quadratic terms; but the effect of nitrogenous fertilisers is not only approximately linear over the range concerned, but since increasingly heavy drainage generally removes decreasing quantities of nitrates, the tendency of its effect is to reverse the curvature of the regression. In the second place it might have been expected that the effects of rainfall would be closely related to the total crop on the different plots, and that resemblance would be more clearly apparent in the proportionate than in the absolute effects; but since for a wide range of manurial condition the absolute effects of additional nitrogen are not very unequal, and tend to decrease with increasing crop, it is apparent that the similarities in the absolute effect, which we observe for example in plots 8 and 14, are more probably due to quantitative

changes in nutrition than to indirect modifications of the physiological condition of the crop.

As will be seen from Table II, the 13 plots studied may be arranged in the following classes :—

(i) Low fertility, no added nitrogen. Plots (3 and 4), 5 and (17 and 18) mineral series.

(ii) Progressively higher yields due to progressive increase in nitrogenous dressings, from plot 6, through plots 7, 13 and (17 and 18) ammonium series, to plot 8.

(iii) Dressings which have produced an increasingly unbalanced nutrition with diminution of yield. Plots 12 and 14.

(iv) Unbalanced nutrition produced earlier and more intensely. Plots 10 and 11.

(v) Farmyard manure containing the very heavy contribution of about 200 lb. of nitrogen, with organic matter which has materially lightened the soil.

An examination of these diagrams shows how intimately the response of the crop to weather is connected with the manurial condition of the soil. Classing the plots solely by inspection of the curves of response to rainfall we shall put together every case in which the manurial treatment is alike, and indeed the whole series of curves arrange themselves in sequence of order of increasing abundance of nitrogenous fertilisers. As preliminary to a fuller discussion we may note (i) that in all the plots the average effect of additional rain is harmful. This agrees with Hooker's finding for Eastern England. (ii) In all save the dunged plot 2B, the average loss is rapidly reduced during August, an observation which finds a simple explanation in the fact that the average date of carting the crop is August 24th, so that as August advances an increasing proportion of years occur in which the rain is too late to affect the crop. The exceptional behaviour of plot 2B indicates that the average loss per inch of rain in the month preceding harvest is even heavier than that shown in the diagram. (iii) In all the plots October is a month in which the average loss per inch of rain is small, or in which rain above the average is positively beneficial. This is the reverse of the condition found by Hooker, who finds the greatest negative correlation with rain early in October. (iv) In all plots save two, the unmanured plot (3 and 4) and plot 5 which receives mineral (non-nitrogenous) manures only, the autumn period of benefit, or but little loss, from rain, is followed by a period centred in January in which dry conditions appear to be particularly desirable.

At this time of the year each additional inch of rain costs from one to two bushels in the crop. That this effect is scarcely visible on the unmanured plot, and still less so on plot 5, speaks strongly in favour of Sir John Lawes's view that the damage done by winter rain was principally occasioned by the washing out of nitrates from the soil. Plot (3 and 4) and especially plot 5 can have little to lose in this way, and possibly rely for the greater part of their growth upon nitrates produced by bacterial activity as the soil grows warmer in March, April and May. The same view is confirmed by the fact that among the remaining plots the loss due to winter rain is least in plots 2B, 10 and 11, all of which are characterised by the low proportion utilised of the nitrogen supplied.

If these plots seldom suffer from lack of nitrogen, its loss during the winter must be expected to affect the crop relatively little; nevertheless the actual average loss of about one bushel to the inch of rain shows that at this time (*i.e.*, before the spring dressings are applied to plots 10 and 11, and perhaps before bacterial activity can fully tap the resources of the farmyard manure), a temporary shortage sometimes injures the crop. With these preliminary observations we may pass to the consideration of the types of rainfall response curve actually found.

Type I.—Plot 5, mineral manures only, and plot (3 and 4), unmanured. Fig. 4 shows plot 5.

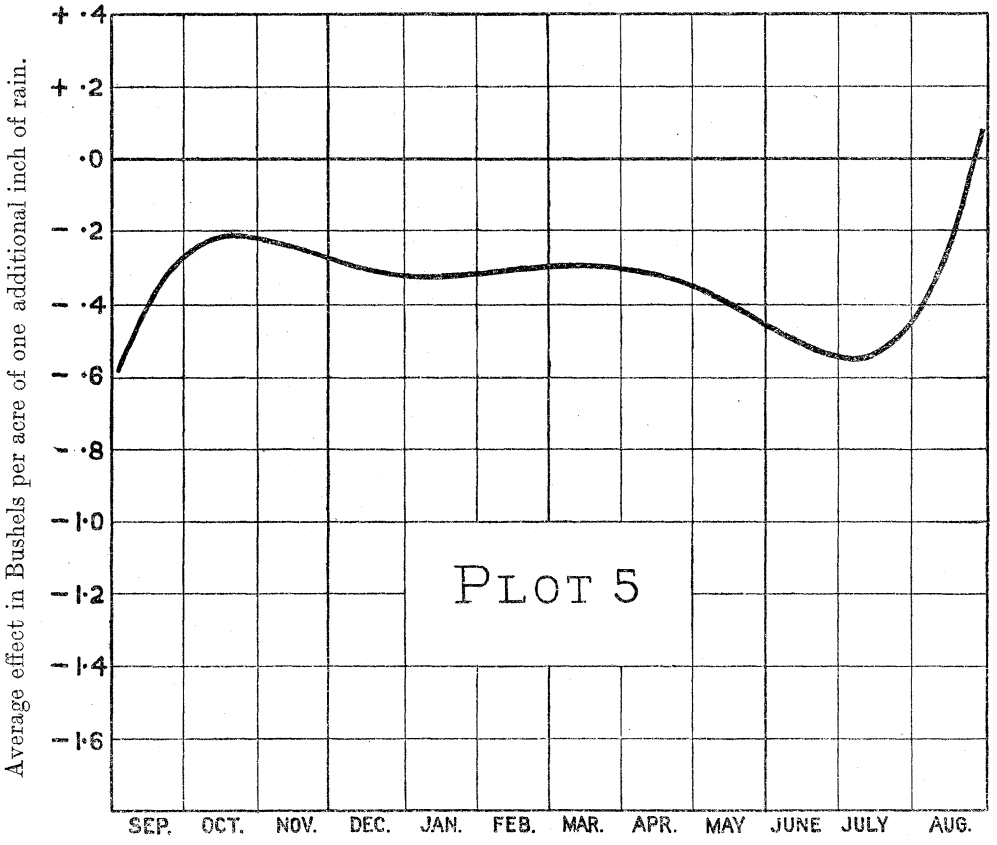


Fig. 4.

In these two plots the crop is severely limited by the lack of available nitrogen. Plot 5 has a mean yield of 14·18 bushels per acre, while plot (3 and 4) yields 12·27. These are averages (1852–1918) covering the whole period over which the rainfall regressions have been calculated. Both plots show heavy deterioration over this period. The small difference in the mean yields, which is nearly constant over the whole period, shows that lack of nitrogen is the dominant limitation in plot (3 and 4) as in plot 5 where the limitation due to lack of nitrogen must be most stringent.

In plot 5 the rain experienced is in excess of the optimum probably at all times of the year. The average loss for additional rain falls from about 0·4, in the September before sowing, to a minimum of 0·22 (1·5 per cent.) towards the end of October, and

remains steadily near to 0·30 till the end of May. It falls to a maximum loss of 0·55 (3·9 per cent.) early in July and rapidly diminishes through August. Since the proportion of drainage through the twenty-inch gauge decreases from nearly 80 per cent. in the winter to about 25 per cent. from April onwards, it follows that if the main loss on this plot due to rainfall is ascribed to the washing out of nitrates, the loss due to an equal amount of drainage is increasingly serious as the year advances. This accords with the belief that nitrification becomes more active in the Spring, and that this plot relies largely for its nitrogen supply upon bacterial activity at this season. The curve even suggests the possibility that under the intense nitrogen starvation of this plot the supply of nitrates is of importance as late as the beginning of July, though it must be borne in mind that other effects of rain may become important during the summer.

The curve for plot (3 and 4) is closely similar. September damage of about 0·4 is followed by a minimum of 0·23 (1·9 per cent.) in October. There is slight evidence of a maximum of damage in the winter of 0·37 (3·0 per cent.) followed by a minimum damage of 0·24 (2·0 per cent.) at the end of March; this feature becomes marked in plots in which the nitrogen limitation is less acute, and is perhaps an indication that this condition is less extreme in the unmanured plot than in plot 5. The maximum damage in early summer is 0·58 (4·7 per cent.); little importance can be attached to the positive values of the last 12 days of August, save as in indication that August rain is inoperative after the first week of the month.

Type II.—Plot 6, complete minerals with single dressing of ammonium salts applied half in Autumn and half in Spring; plot 17 and 18 mineral series, receiving mineral manures only, but alternating with the ammonium series which receives a double dressing of ammonium salts. Fig. 5 shows plot 6.

The similarity of the curves for these two plots is striking, and requires that the interpretation put on the yields of the alternating plot should be reconsidered. Plot 6 has a mean yield of 22·58 bushels to the acre, and shows a relatively rapid deterioration; 17 and 18 minerals has a mean yield 14·51, scarcely more than that of plot 5 which receives the same dressing; its average deterioration also resembles plot 5. Hence it has been thought that no appreciable benefit accrued to the alternating series from the previous year's dressing of ammonium salts. It has however been observed (5, 1921) that the alternating series is much more variable in yield than is plot 5, and this suggests that the additional variation is due to variable residue of nitrogenous material. This suggestion is strongly confirmed by the resemblance to plot 6, especially in the effects of winter rainfall.

Plot 6 has a strongly marked minimum of damage in early October, when the loss is only 0·10 bushel per inch of rain, or 0·4 per cent.; this is followed by a winter period in which rain is on the average particularly harmful to the extent of 1·14 (5·0 per cent.); from April to July the average damage is nearly constant at about 0·64 (2·8 per cent.).

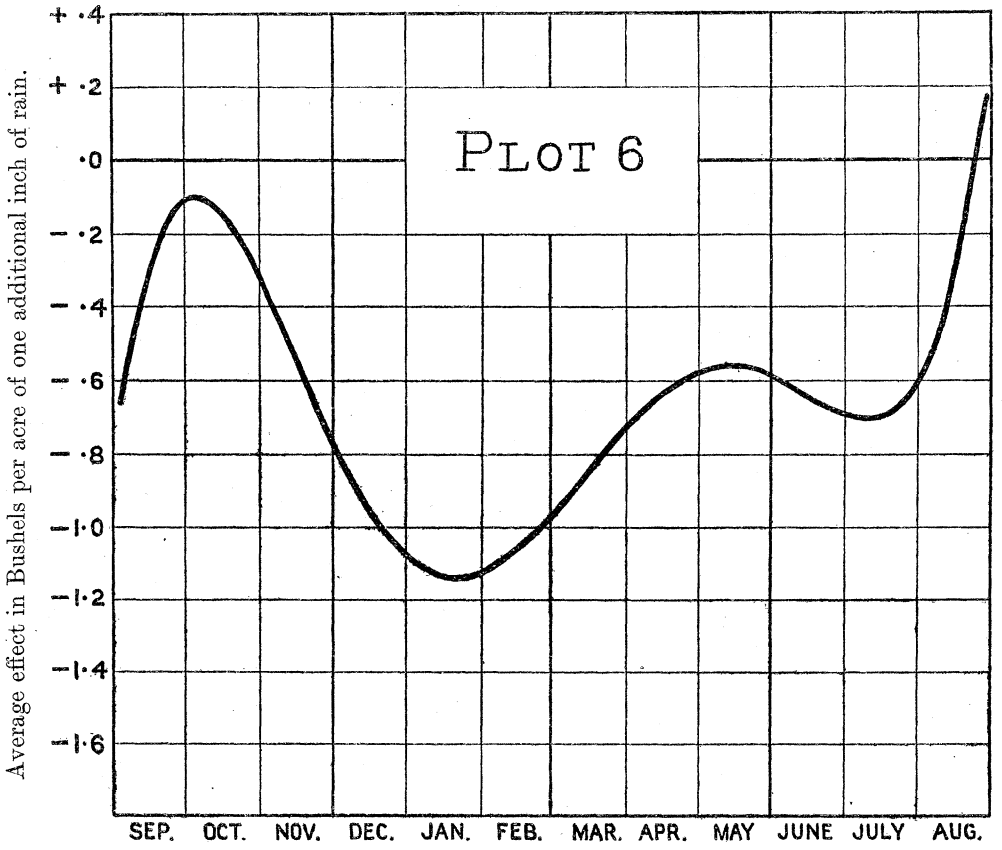


Fig. 5.

The effects on 17 and 18 minerals are, on the whole, smaller, being approximately proportional to the lower yield. A minimum of damage occurs in October 0·10 (0·7 per cent.), followed by a winter maximum 0·64 (4·4 per cent.), and by a nearly stationary period from April to July at 0·42 (2·9 per cent.). The winter effect is smaller both absolutely and relatively than that of plot 6, but if it be admitted that the importance of winter rain lies largely in the washing out of nitrates, the evidence for considerable, though no doubt variable, residues in the alternating series is unmistakable. The fact that the alternating series has not given an appreciable higher yield than that of plot 5, must be ascribed to soil heterogeneity, which has approximately balanced the advantage of the residual nitrogen. That such soil heterogeneity exists in this field may be shown by a comparison of 2*a* with the adjacent plot 2*b*. These have received the same manure since 1885, but during the present century 2*a* has ceased to gain in yield upon 2*b*, but yields very regularly on the average 2 bushels less than the adjacent plot. In view of this fact we cannot deny the possibility that with equal manuring the land of plots 17 and 18 would yield 2 or more bushels less than that of plot 5, and that this circumstance has served to mask the advantage accruing to the mineral series owing to residual nitrogen. It may be mentioned in addition that the mean yield of 17 and 18 ammonium series is 2·36 less than that of plot 7, a difference which may be ascribed to the combined effect

of soil inferiority and lack of residual nitrogen; it should be borne in mind that a difference of this amount in a yield of 30 bushels probably would require the addition of considerably more nitrogen than an equal increase in a yield of 14 bushels.

Type III.—Plot 7, complete minerals, with double dressing of ammonium salts, of which one quarter is applied in autumn.

Plot 13, as plot 7 without sulphates of sodium and magnesium.

Plot 17 and 18 ammonium series.

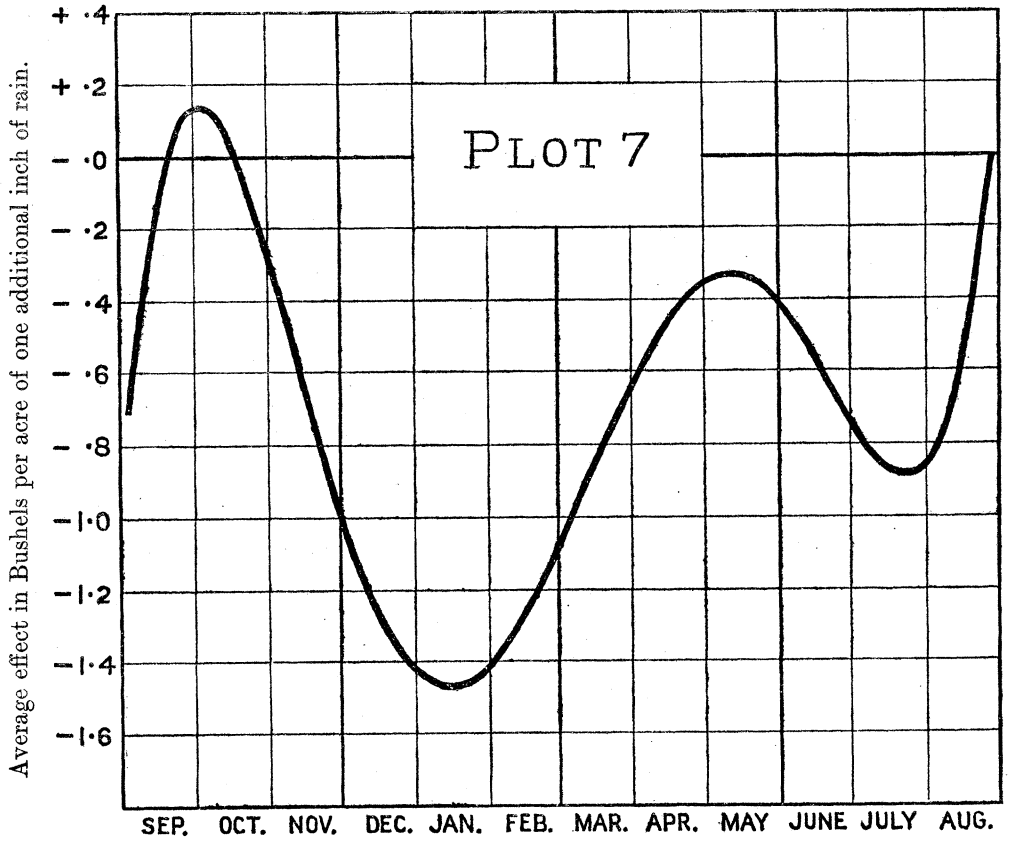


Fig. 6.

These three plots receive similar manurial treatment; the mean yields are 31·37, 30·21 and 29·01 bushels per acre, with mean annual decrements 0·144, 0·123 and 0·114. It might have been anticipated that the decrement should be least on the alternating plot; for the rest it is not obvious to what circumstances the small difference between the plots are to be ascribed.

The three rainfall curves are generally similar, all indicate a slight benefit for something over a month in autumn, the maxima being + 0·14 (0·4 per cent.), + 0·43 (1·4 per cent.), and + 0·41 (1·4 per cent.); the winter damage is strongly marked with values 1·47 (4·7 per cent.), 1·82 (6·0 per cent.) and 1·85 (6·4 per cent.). There is in all cases a clear period of minimum damage in May, the values falling to 0·34 (1·1 per cent.), 0·34 (1·1 per cent.) and 0·20 (0·7 per cent.), followed by a second period of maximum damage in July which is however less severe than the winter maximum, the values are

0·89 (2·8 per cent.), 0·79 (2·6 per cent.) and 0·94 (3·2 per cent.). On the average plot 13 suffers somewhat more heavily from rain than do plots 7 and the ammonium series, but in considering the effects at different times of the year it is seen to agree with the latter closely during the first half of the year, and more nearly with plot 7 during the second half.

Type IV.—Plot 8 as plot 7 with additional ammonium salts applied in the spring.

Plot 12 as plot 13 with substitution of sodium for potassium sulphate.

Plot 14 as plot 13 with substitution of magnesium for potassium sulphate.

Fig. 7 shows plot 8.

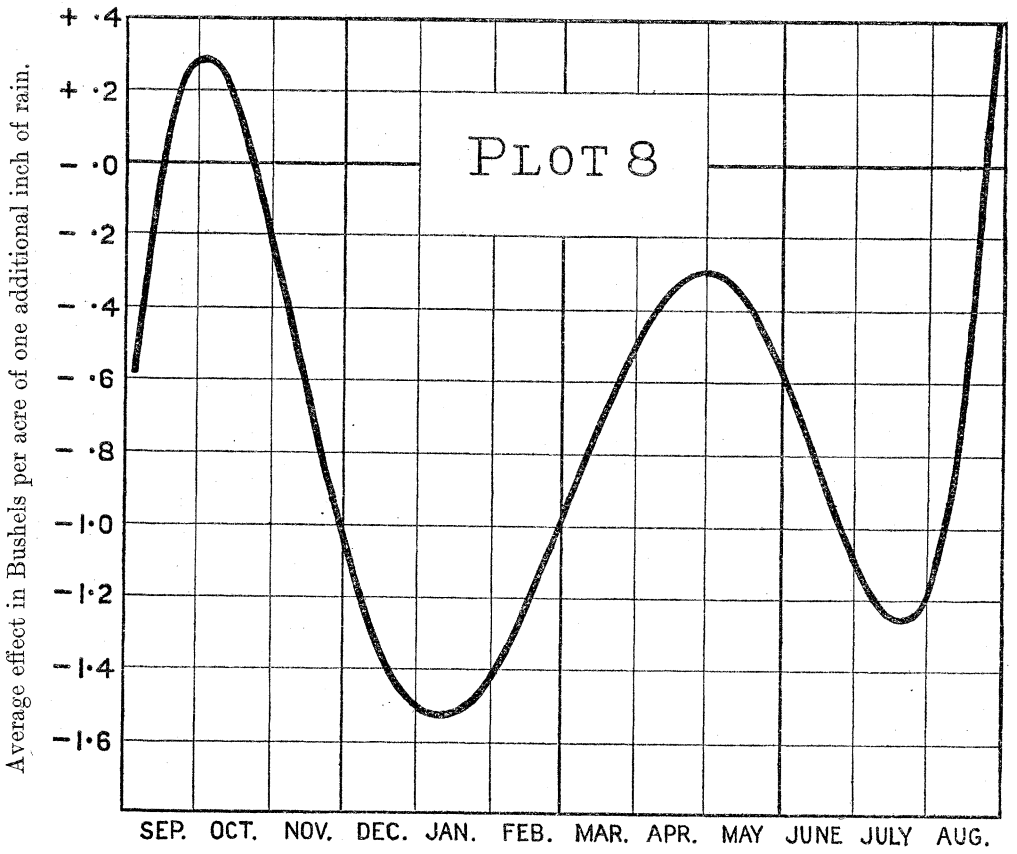


Fig. 7.

In most respects plot 8 contrasts strongly with plots 12 and 14, which however it resembles in its response to rainfall. The mean yield is 35·69 against 28·32 and 27·77, while it shows but slight deterioration, 0·092 annually, as against the heavy deterioration, 0·181 of plot 12, and 0·231 of plot 14. The resemblance consists in the greater relative abundance of nitrogen supplied to these plots than to those in Type III, and the correspondingly less relative abundance of nitrogen than those in Type V. In the case of plot 8, although the yield is greater than that of plot 7, yet since it receives a triple in place of a double dressing of ammonium salts, it is clear that this plot must less frequently suffer from lack of nitrates. Plots 12 and 14 on the other hand receive the same nitrogenous dressing as plot 7, but owing to lack of potash yield some 3 bushels an acre less ;

consequently with them also crop limitation through lack of nitrates must be less frequent. The same argument applies still more strongly to plots 10 and 11 in the next type.

The benefit of October rain is relatively large in plots 8 and 14, and plot 12 also reaches positive values at this period, the three values are + 0·29 (0·8 per cent.), +0·10 (0·3 per cent.) and + 0·36 (1·3 per cent.); the winter damage is strongly marked, though on the whole less so than in the preceding group, 1·53 (4·3 per cent.), 1·65 (5·8 per cent.), 1·52 (5·5 per cent.). The chief contrast lies in the deepening of the second period of maximum damage; in consequence of this the time of minimum summer damage is shifted back to the beginning of May with values 0·30 (0·8 per cent.), 0·49 (1·7 per cent.) and 0·23 (0·8 per cent.); while the second maximum reaches in late July the values 1·27 (3·5 per cent.), 1·48 (5·2 per cent.), 1·23 (4·4 per cent.), showing that dry weather at this period is to these plots nearly as beneficial as in January.

Type V.—Plot 10 double dressing ammonium salts only.

Plot 11 double dressing ammonium salts with superphosphate. Fig. 8 shows plots 10 and 2B.

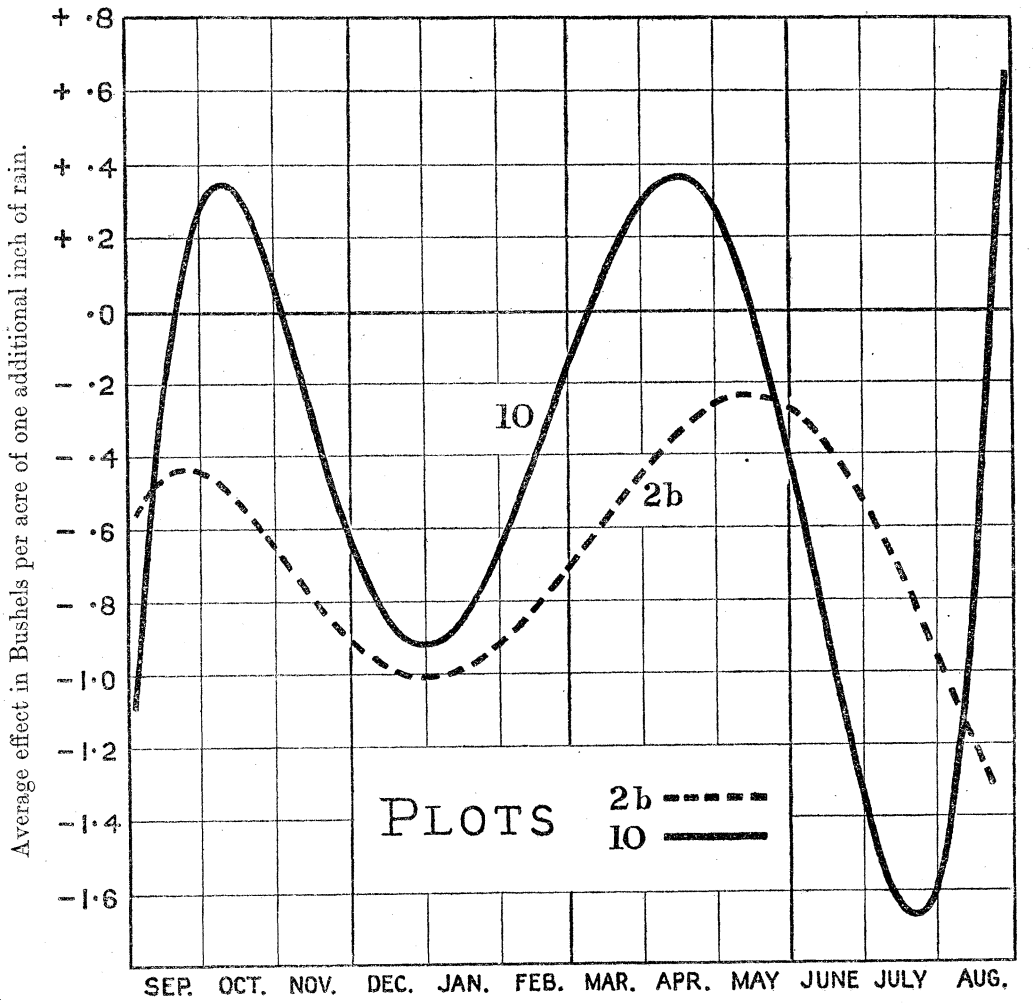


Fig. 8.

In these two plots the yield is very seriously lowered by lack of potash, and in the case of plot 10 by lack of phosphate also. The mean yields are 19·50 and 22·05, with heavy annual deterioration 0·157 and 0·219. Plot 10 with the lower yield is, as might be expected, the more extreme example of the type.

Both curves are positive in October with values + 0·35 (1·8 per cent.) and + 0·16 (0·7 per cent.); the winter damage is less marked in absolute value than in the types III and IV., being only 0·92 (4·7 per cent.) and 0·85 (3·8 per cent.); in April positive values are reached in both cases, those of plot 10 being even higher than in October, + 0·37 (1·9 per cent.) and + 0·03 (0·1 per cent.). The July damage is more extreme than in preceding types being considerably greater than that in January, namely, 1·67 (8·5 per cent.) and 1·43 (6·5 per cent.). In both the April and the July values plot 10 contrasts more strongly than plot 11 with the plots of type IV.

Type VI.—The dunged plot 2B (fig. 8).

This plot differs from all the others in the texture of its soil, and to this perhaps may be ascribed the absence of strong contrasts in the effects of rain at different seasons; the average advantage of dry weather is comparable with the other plots, but the only marked feature of the curve is the severe damage caused by rain immediately preceding harvest. This feature may perhaps be analogous to the heavy damage of July rain in plots 10 and 11, which it should be noted resemble the dunged plot in their relative abundance of nitrogen. Plot 2B differs strikingly from all the other plots, and especially from plots 10 and 11 in the relative constancy of its yield.

The mean yield is 34·55 bushels per acre. The autumn minimum of rain drainage is 0·43 (1·3 per cent.), the winter maximum 1·00 (2·9 per cent.), the summer minimum is well marked and prolonged at 0·24 (0·7 per cent.), after which the curve falls to a final value of 1·39 (4·0 per cent.); as we have seen, the mean damage for harvest rain which catches the crop is no doubt greater.

10. *The Value of Rainfall Regressions as Prediction Formula.*

The extent to which the variation in crop yield is predicable from a rainfall record may be calculated from the coefficients of multiple regression of the last section. As we have seen in section 2 the coefficient of multiple correlation gives a much exaggerated notion of the prediction value of a regression formula, if calculated from a small sample. In the present instance, although our series of values is a long one, the number of degrees of freedom has been whittled down to 54 by the rejection of unsuitable years, and the elimination of slow changes; six meteorological variates have been used, which represents the utmost economy in view of the complexity of the meteorological sequences. The distribution of the variance, between the 6 degrees of freedom of the regression formula, and the 48 degrees of freedom in which the variates may differ from the regression formula is shown in Table XIV, in which are also shown the multiple correlation R, and the percentage of variance (A_1 of section 2) ascribable to the average effect of rain.

TABLE XIV.

Plot.	Sum of squares.		Total 54 degrees.	R.	A_1 per cent.
	Regression formula 6 degrees.	Deviations 48 degrees.			
2B	884	996	1,880	0·6859	40·42
3 & 4	149	336	485	0·5548	22·13
5	163	401	564	0·5370	19·93
6	563	1,062	1,625	0·5885	26·46
7	790	2,091	2,881	0·5236	18·34
8	959	2,083	3,042	0·5614	22·96
10	796	1,827	2,623	0·5509	21·63
11	668	2,501	3,169	0·4592	11·23
12	1,395	2,069	3,464	0·6345	32·79
13	985	1,938	2,923	0·5803	25·39
14	938	2,069	3,007	0·5584	22·58
17 & 18 M.	199	686	885	0·4736	12·73
17 & 18 A.	965	1,189	2,154	0·6693	37·90

It will be seen how very inadequate is the value of R to indicate the value of prediction formula; the extreme values of R in the above totals are 0·459 and 0·686, but these values indicate in this case that in one plot 11 per cent., and in the other 40 per cent. of the variance, is expressible in terms of the sequence of rain records.

It is remarkable that so much of the variance as 40 per cent. should be expressible in terms of a single meteorological element such as rainfall, especially when it is remembered that all causes of variation without exception, including casual errors, and the quadratic terms of the rainfall effect, are included in the remaining 60 per cent. This leads us to think that a record of rainfall, in spite of the many disabilities which have been urged against it, is of more value than the record of any other single element, in characterising the season.

The effects ascribed to rain are in most plots clearly significant; the values of P, calculated from the formula

$$(1 - R^2)^{24} (1 + 24R^2 + 300R^4)$$

range from 0·0000186 to 0·0659.

The excessive variation of plot 11 thus masks the rainfall effect in the same manner that it masks the slow changes in this plot; the rainfall effect is somewhat the more important and shows up more clearly. In both cases the similarity of the curves of plot 11 to those of other plots, especially those which it would be expected most nearly to resemble, shows that no serious deviations have been introduced into these by random fluctuations.

The probable errors of random sampling of the regressions of crop on rainfall may be calculated as demonstrated by the author in 1922 (9). The number of years is sufficient to ensure the effective normality of the distributions. In the comparison of the regression, of any two plots, we may anticipate that the random errors are on a substantially smaller scale, since the experience of all the plots is drawn from an identical series of seasons.

No statistical estimate of the accuracy of curves for purposes of comparison can be made in the absence of strictly parallel plots. It may, however, be confidently anticipated that random errors of this kind are, if anything, of a smaller order than the differences between the curves obtained for plots 7, 13 and (17 and 18) ammonium series.

Reality of Slow Changes.

The prediction formulæ which we have obtained are entirely independent of the slow changes which appear to have taken place in yield and in weather. In presenting the evidence for slow changes in yield (5) it was presumed, in the absence of a full investigation of the sequence of rainfall records, that favourable and unfavourable weather conditions fluctuated independently from year to year; in fact, that slow changes were absent from the meteorological series. We now know that a small percentage of the variance of annual rainfall should probably be ascribed to slow changes, and that sequences of wet years did in fact occur about 1879 and 1914. Since these two periods agree with the two main depressions in yield, it becomes at first sight questionable whether the slow changes in yield may not after all be ascribable to meteorological effects.

This question is most readily answered by calculating the actual depression in yield ascribable to the additional rainfall in the two rainy periods. Plot 2B is most suitable for a comparison for the residual variance is, in comparison to the yield, very small in this plot, and the prediction formula correspondingly accurate. The yield in any year is regarded as made of two parts, part (positive or negative) is due to the rainfall sequence under which the crop was grown, the remainder is the yield handicapped for the advantages or disadvantages ascribable to rainfall. The sequence of 10-year means of these two quantities is shown in fig. 9. It will be seen that the yield, after making allowance for rainfall, still shows strongly the slow changes which originally attracted attention. The changes in weather only account directly for a portion of the slow changes observed; in the first half of the sequence this portion is roughly one-half, but in the latter half it would appear not to exceed one-quarter of the total effect.

The circumstance that in two cases the depression of yield, after allowing for rainfall, coincides with a series of years in which the rainfall was on the average unfavourable, is in accordance with the suggestion previously put forward, that in addition to the immediate effect of rainy weather on the growing crop, a sequence of wet years produced an additional and prolonged unfavourable influence by fostering weeds. Two cases of agreement are, however, quite insufficient to prove that the additional depression of yields is causally connected with the rainy periods. The fact of the coincidence, whether causal or fortuitous, does, however, emphasise the importance of eliminating slow changes in the study of annual figures; if slow changes had been ignored in carrying through the correlational work, we should no doubt have arrived at greatly exaggerated estimates of the harmful effect of rain. Higher correlations would have been obtained and the results would have exhibited one more case of high but essentially unreliable correlations in meteorological agriculture. Against the view that the depression in yields is an indirect effect of a rainy period must be set the fact that in our diagrams the yield appears

to fall off somewhat *before* the wet sequence has set in ; both series being 10-year means the slight falsification of the sequence incurred by using this form of representation should have a similar effect in both series.

Effect of Fallow.

Certain years were rejected owing to the crops grown having followed a fallow ; it was suspected that the yields obtained in these years were unduly high, and would vitiate

Fig. 9.—Graph A shows the variation in the expectation of wheat yield as judged from the quantity and date of the rain during the year. Graph B shows the additional variation of the actual yields observed after allowance for the expected rainfall effect. Both graphs show 10-year means, in bushels per acre.

the weather correlations if they were retained. A knowledge of the expected effect of weather, together with that of the general fertility of the plot in neighbouring years, makes it possible to estimate the amount by which the fallow benefited the crop. On plot 2B the excess of yield over expectation in 1890, 1891, was 4·48, 6·77 ; in 1905, 1906, the excess values were 6·98, 5·95, while in 1915 the excess was nearly 15·50. In 1916 the two halves of the field were harvested separately, and the half that followed fallow showed an excess of 14·30 bushels over the other. This suggests that the value for 1915 is not so inaccurate as might be thought.

In all these 6 cases the effect of fallow is strongly marked ; the average gain is the very large amount of 9 bushels per acre. It should be remembered that this plot receives a heavy dressing of farmyard manure, and it is doubtful if any part of the gain can be ascribed to the accumulation of plant nutrients. The immediate object of fallowing was in all cases the eradication of weeds, and it may be that the increased yield was mainly due to a greater freedom of the land from weed infestation. If that is so the effect must

have been remarkably transient, for the value for 1916 depends on a comparison of the crop immediately following fallow with one which had been fallowed only two years before. The supposition that the benefit was largely a matter of freedom from weeds accords with the higher values of 1915, 1916, when the field, to judge by the average yields on all plots, was much more severely infested than in 1890 or 1905.

The magnitude of the effect of fallow illustrates the complications which arise in the interpretation and statistical reduction of even the best agricultural data. We have attempted to eliminate the large changes which have occurred in the yields of these plots by means of continuous curves, but there is reason to fear that even if the fertility of the soil normally suffers continuous changes from year to year, yet discontinuities were introduced at the periods when fallowing was resorted to. By rejecting the crops immediately concerned the greater part of the errors have no doubt been eliminated, yet the theoretical efficacy of the continuous curve is impaired, however nearly it appears in practice to represent the actual course of events.

11. *Comparison with Previous Results.*

In 1880 LAWES and GILBERT published (12) a long and careful account of the meteorological characteristics of seasons favourable and unfavourable to wheat. The object was to effect a qualitative and preliminary enquiry (p. 174): "As yet, however, the connection between meteorological phenomena and the progress of vegetation is not so clearly comprehended as to enable us to estimate with any accuracy the yield of a crop by studying the statistics of the weather during the period of its growth. . . . . But it is only by a careful comparison of the characters of the seasons on the one hand, and of the quantity and quality of the produce on the other, for many years, that we can hope to acquire sufficient knowledge to enable us to assign to the various agencies, the sum of which constitutes the climate of the year, their respective values in the production of the crop."

In respect of rainfall, LAWES and GILBERT, using principally Rothamsted data, but also some instances of years of exceptional harvests prior to their experiments, concluded that comparative dryness was desirable from Seed-time (November) to harvest, especially in Winter and early Spring. Their table of averages shows no exceptional dryness of October for the favourable years, or exceptional wetness in the unfavourable years. The effect of winter and spring rain is ascribed partly to drainage causing loss of nitrates, and partly to hindering root development.

It appears that in the points upon which they laid most stress the conclusions arrived at by the more exact statistical methods now available and with the aid of 39 more seasons' experience at their station, would have caused LAWES and GILBERT only to reaffirm their conclusions more strongly and with greater precision. The comparison of different plots has emphasised the importance of the drainage of nitrates, and points to further influence of manurial conditions on the response of the crop to July rain. The cause of this is at present obscure, but the effect seems well marked. In addition it has

been possible to make some attempt to develop formulæ which shall predict the yield from the weather statistics.

Attention was called to the possibility that autumn rainfall was an important factor in determining the wheat crop by Sir NAPIER SHAW in 1905 (14). SHAW did not use the method of correlation, but pointed out that in the twenty years 1885–1904, with two exceptions, when the yield for Eastern England was above the average the previous autumn rainfall was below the average and *vice versa*. In this set of 20 pairs of values the correlation is actually $-0{\cdot}629$, a value which would only be exceeded by uncorrelated variates in 3 samples of 20 out of a thousand. The particular meteorological variate, autumn rainfall, was picked out of a table giving at least 36 meteorological quantities, and the chance that all of these, if independent and in reality uncorrelated with the crop, should give correlations between $\pm 0{\cdot}629$, was therefore about $0{\cdot}89$. Such a system would therefore be expected to yield so high a correlation only once in 9 trials, and the fact that such a correlation occurred supplied some presumption that autumn rainfall had in reality a perceptible influence on the crop.

In discussing the significance of this result SHAW (14, pp. 318–319) made the interesting suggestion that the association observed might not be due wholly to the effects of autumn weather, but possibly to dry autumns being frequently followed by a favourable succession of weather in Spring and Summer.

> "He would like to say a word with respect of Mr. THOMAS's suggestion that the autumn rainfall was not the dominant factor in determining the subsequent yield of wheat. What surprised him were not the exceptions, but the agreements. Remembering that nine months had to run between the end of autumn and the beginning of harvest, and considering the influence of the intervening rainfall, sunshine and other accidents that might happen to the crop before it was gathered, it was surprising that the connection should be so close as to be expressed possibly numerically. It might be true that two other columns of figures might be tabulated which would show a closer agreement than the two columns put down in the table, but they certainly would not be columns of figures for individual elements. To take two elements out of the whole table, put them side by side, and find them to agree as they did in this case, was astonishing. Of course other influences affected wheat, and it might be that a sequence of influences was required to follow the autumn rainfall in order to bring out the corresponding result. It might be that the relation was a meteorological one, and that a dry autumn itself implied a dry spring or a dry summer, or whatever combination of circumstances was required for a good yield. He could give a certain amount of evidence in favour of the contention that a meteorological relation existed, and that it was not what took place in the autumn alone which might account for the relation, but succeeding events as well, which were associated with a dry autumn. That contingency made the subject one of considerable interest, and one which must be pursued rather more fully than was possible on the present occasion."

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY B BIOLOGICAL SCIENCES

A great advance in method is shown by the magnificent paper by HOOKER (10) in 1907. HOOKER systematically correlated rain and accumulated temperature for 8-week periods throughout the year with the yields of a number of farm crops in Eastern England, using a slightly different area from that used by SHAW. HOOKER clearly recognised the two limitations from which the method of correlation suffers, namely that it takes account only of the *linear* relation of the variates, and secondly that the correlations obtained will be much affected by any such meteorological associations between the weather at different times of the year as were suggested by SHAW. It was to remove these two limitations from the treatment of the Rothamsted data that the present method of computing the partial regressions of the yield on the weather at each period of the year was devised, a method which as we have seen can be extended to the discussion of the quadratic terms of the regression function.

HOOKER found high negative correlations between the wheat yield and the rainfall for periods centred in October and January, while between them small positive correlations occurred. In May again positive correlations, this time somewhat larger, made their appearance. It is concluded that a dry September-October ranks first among the wheat's requirements, while the effect of winter rain is ascribed partially to the high correlation (+ 0·6) found between the rainfall of weeks 1–8 and that of the preceding weeks 37–44.

In 1922 HOOKER (11) gave recalculated figures for the same variates; the results being now based on 35 years, possess considerable significance. Diagram 10 shows the correla-

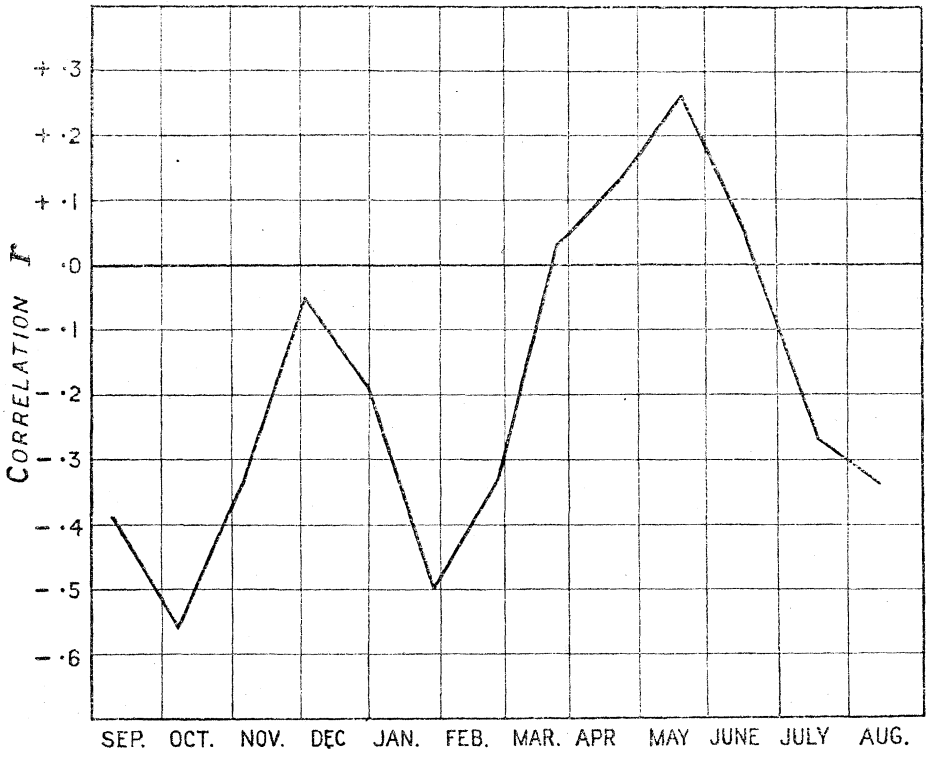


Fig. 10.—Rainfall correlations obtained by HOOKER from official returns of wheat yields for Eastern England, using rainfall for 8-week periods centred at the dates shown.

tions obtained for successive (overlapping) 8-week periods, in respect of which HOOKER draws the following conclusions (p. 120) :—

> "Looking first at the curve showing the connection between rain and wheat, the most striking features are the dips about October and in winter, both very nearly identical. As a matter of fact the winter dip is slightly greater than the other. But no weight can be attached to so slight a difference, and we must apparently regard a dry seed-time and a dry subsequent winter as about of equal importance. Sir NAPIER SHAW appears to have been the first to call attention to the great advantage of a dry autumn, and my calculations fifteen years ago pointed to this period being more important than the winter. It is not very clear from the writings of GILBERT and LAWES how far they realised this feature of the wheat's requirements ; it would seem that they were aware of it, but they lay all stress upon the winter rainfall and the washing of the nitrates out of the soil. Wheat is a deep-rooting plant, and I believe it to be probable that, if experiments could be devised to test the point, it would be found that the real effect of a very wet autumn is mechanical—that it is then more the clogging of the soil that prevents the proper development of the root system than the absence of sufficient nitrates. The effect of the loss of the latter is probably more felt later on, during the winter, after the plant is established, and this is reflected in the heavy negative coefficient with rain during the winter months. GILBERT and LAWES have laid so much emphasis upon the washing out of the nitrates that it is generally overlooked that they did in fact recognise the hindrance to root development caused by saturated soil. It seems reasonable to conclude from the data that the mechanical effect of the autumn is just as important as the chemical effect of the winter ; and it is, moreover, one that is practically irremediable."

The statement that the winter dip is slightly greater refers to the partial correlations obtained after eliminating the effects of temperature ; we are here only concerned with the total effects, and it is apparent that HOOKER'S data contain substantial evidence for a real deleterious effect of Autumn rainfall. In other respects the series of correlations is not out of harmony with several of the Broadbalk plots, the characteristic period of winter damage is clearly recognisable in both, and we may identify the positive correlations obtained in late Spring with the minimum of damage observed on several of our plots at the same season. The July values also are suggestive of the more highly nitrogenous plots on Broadbalk. The fact that HOOKER'S series appears to incline on the whole to more positive values than are observed in our regressions is not improbably due to the inclusion in his data of yields from lighter lands, more susceptible to summer droughts than is the heavy loam of Broadbalk ; in addition his mean rainfall is some four inches less than in our series.

The earlier half of the series, in which the Broadbalk plots are most alike, is alone in contradiction to HOOKER'S series. Of course, if strong meteorological correlations existed between the several parts of the year, no resemblance need be expected between

the actual regressions and the correlations which are, in a complicated manner, compounded from them; and even slight meteorological correlations might seriously disturb the numerical values of the correlations with the crop; but our meteorological data show that such correlations between weather at different periods of the year, though probably present, are quite small; and since the agreement, in its general qualitative outline, has not been disturbed from January onwards, it is probable that other factors contribute to the striking difference observable in the effects of October rain. In particular it may be noted that HOOKER's values at this period, allowing for the fact that adjacent points are not independent, but have four identical weeks in common, show somewhat abrupt changes; the values found for November-December agree well with our maximum in October, and the damage ascribed by HOOKER to "seed-time" may be paralleled by the somewhat increased damage shown in many of our curves in September. In this connection an investigation is desirable of the accuracy with which HOOKER's process of correlating with sets of 8-consecutive weeks, is able to indicate the true maxima or minima of the correlation curve; and to what errors such estimates are exposed owing to the capricious incidence of heavy rain.

A consideration which more probably contains a solution of the discrepancy is that under industrial farming conditions rain in late autumn and early winter prevents the sowing of large areas of wheat, the land being sown in spring with oats or barley. This is a factor which must strongly influence HOOKER's results, whereas under experimental farming conditions it is inoperative. HOOKER mentions that the wheat area has a correlation $-0{\cdot}41$ with rain of weeks 37–44, but does not mention the correlation of neighbouring periods; these it would be necessary to know in tracing out the influence of the variable wheat areas on the yield.

It would seem probable that the proportion of land lost to wheat in this way should differ from district to district, and even more probable that such loss should not be proportionately distributed between the two main classes of wheat land in which the wheat follows respectively clover and roots. There are thus many points at which the effect of autumn rainfall upon cropping might give rise to an apparent influence on yield.

It is admittedly by no means necessary that the response of crop to weather should be the same on one particular type of land as it is on the average wheat land of several countries; the curve of regression may be as much influenced by soil type, as we now know it to be by the manurial condition of the soil. In this respect data of crop averages grown under industrial conditions would be of the greatest value in supplementing by shorter series over a more extensive area the long series from a single field which the Broadbalk data provide. It is to be feared, however, that the Ministry of Agriculture's returns are not sufficiently accurate to meet this need, being based principally upon eye estimates of yield before harvest. Such estimates may be closely correlated with the true yields, and yet fail to give the values of the required regressions even with approximate accuracy; for there is reason to fear that the deviations from the known mean yield are systematically, though of course unconsciously, underestimated. In

inquiries respecting the weather it would be important to ascertain also that the known meteorological peculiarities of the year under review, the supposed effects of which upon the crops may have been discussed in the press, are without systematic influence upon the judgments of reporters.

12. *Summary.*

The study of the theoretical distributions of statistics emphasises the dangers of applying the methods of multiple correlation to small samples, and the necessity of extensive crop data in the study of meteorological agriculture.

By a special procedure involving the analysis of separate meteorological sequences it is possible to obtain an adequate mathematical expression of the average effects of the meteorological influences indicated by different instrumental observations at different times of the year.

The errors involved in the correlation of residuals of series changing in an unknown manner may be minimised by the method of polynomial fitting ; such errors are probably insignificant when this procedure is applied to rain data and wheat yields.

The rain data for Rothamsted have been analysed for 65 years ; there are some indications that the wet years tend to occur in spells ; a continuous and progressive change is observable in the distribution of rain through the year ; in other respects the sequence appears to be fortuitous. Rainfall changes account for only a portion of the slow changes observed in the yields.

Curves showing the average effect on the yield, for each additional inch of rain, throughout the year, have been obtained for 13 plots of Broadbalk wheat field, which have been under uniform experimental treatment since 1852. On all the plots dry weather is generally beneficial. A detailed comparison of the several plots indicates a predominant influence of the effect of rain in removing soil nitrates ; the cause of other well-marked features cannot safely be asserted without further research, which it is hoped may be facilitated by the body of facts expressed in these curves.

Previous investigations bearing particularly on the present data are briefly discussed in the last section.

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